UNIVERSITY OF PENNSYLVANIA

CHANGES IN THE CONTENT OF ELEMENTARY ALGEBRA

SINCE THE BEGINNING OF THE HIGH SCHOOL MOVEMENT AS REVEALED BY THE TEXTBOOKS OF THE PERIOD

AMY OLIVE CHÂTEAUNEUF

A THESIS

IN EDUCATION

PRESENTED TO THE FACULTY OF THE GRADUATE SCHOOL IN
PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR
THE DEGREE OF DOCTOR OF PHILOSOPHY

PHILADELPHIA
1929

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AMY OLIVE CHÂTEAUNEUF

PHILADELPHIA
1929

DEDICATED

TO

MY MOTHER AND MY SISTER

WHOSE SELF-SACRIFICE

AND LOYAL DEVOTION

HAVE MADE THIS

STUDY POSSIBLE

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PREFACE

Elementary algebra has been for many years an important subject in the high school program of studies. However, there have been many discussions concerning just what this subject should include. A few years ago the National Committee on Mathematical Requirements proposed changes in content, in methods, and in organization. These changes, which are being more or less adopted by an increasing number of schools, appear to some too revolutionary and to others not sufficiently so. To what extent the committee has actually broken away from the traditional high school course, however, or how far back the present-day tendencies extend, cannot be determined until, among other questions, the following is answered: "What was taught in elementary algebra in the past, not just in the immediate past, but approximately since algebra has been a high school subject?" The present study attempts to answer this question as far as such an answer is contained in the contemporary algebra textbooks of the various decades since the inception of the high school movement.

The writer is greatly indebted to those whose aid has made this study possible. The members of the School of Education, University of Pennsylvania, collectively and individually, were of great help. For the ever-present inspiration, guidance, and help of Dean John H. Minnick, chairman of the thesis committee, the writer is especially grateful. Likewise she wishes to thank and gratefully acknowledge the assistance of the other members of the committee, Professors A. Duncan Yocum, E. Duncan Grizzell, and W. Thomas Woody. An expression of appreciation is due also to those—too numerous to mention all by name—who have rendered cordial assistance in connection with various parts of the work. In this respect acknowledgment is made particularly to Dr. Lao G. Simons of Hunter College, whose bibliography of American algebra textbooks proved invaluable. A great debt of gratitude is due to the writer's mother whose encouragement and inspiration never wavered, to her sister who constantly helped with the many details of the work, and to her father, whose valuable cooperation permits the prompt publication of this study.

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INTRODUCTION

Nature and Purposes

This study is historical in nature. It has for its purposes:

1. To determine the changes which have developed in the content of the topics of elementary algebra[1] approximately since the beginning of the high school movement, as revealed by the contemporary textbooks of the various decades.

2. To compare these changes and set forth the central tendencies shown to have existed during this period.

3. To present the whole in a form conducive to viewing present and future changes from a correct perspective.

Scope and Sources

This study deals only with the content of the topics of elementary algebra as revealed by the textbooks in the subject. It does not attempt to tell how this content was taught or why it is so, as these, although closely related to content, seem large enough in themselves to be considered separate problems.

The source of material is limited to the textbooks in the subject. In the first place, an intensive study was desired. In the second place, textbooks probably form the most important single index to what was actually taught in the past. In the third place, the material from this source seemed sufficiently extensive in itself to lead to reliable results.

Method

The first step consisted in the selection of appropriate books. Textbooks, copyrighted or registered between 1818 and 1928 inclusive, on elementary algebra as previously defined and

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[1] Elementary algebra is defined for this purpose, as the first course in algebra in a high school; academy of equivalent rank; or, in the earlier decades, the higher department of a grammar school, as this often represented the beginning of a high school. In other words, it is the course now commonly given in the ninth school year.

adapted to or published for American schools were chosen.[2] Books for special types of schools or books on general mathematics were omitted. Different editions of the same book were not used unless revised recopyrighted, or re-registered. The result was a total of 257 books by 158 different authors. This probably constitutes a large sampling of the elementary algebra textbooks published during this period[2, 3]. The books were then classified by decades according to date of copyright or registration. The distribution for each decade was as follows: 1820, 10 books; 1830, 14 books; 1840, 20 books; 1850, 24 books; 1860, 9 books; 1870, 17 books; 1880, 20 books; 1890, 33 books; 1900, 44 books; 1910, 31 books; 1920, 35 books.

The second step was the determination from these books[4] of the broad, general topics of elementary algebra. It was found that until recently elementary and more advanced algebra were often published together. Some books drew no dividing line between the two; others drew a line of demarcation more or less distinct and recognizable. It became clear, therefore, that an arbitrary selection of elementary topics must be made. Hence, all topics appearing in more than a third of the books that made a division between elementary and advanced algebra and that were written by different authors, were arbitrarily chosen as the most likely topics in elementary algebra for that decade. The percentage was put as low as this so that any topics just beginning to gain general favor would be included in the study; it was put as high as this so that any advanced topics put in to make the books appeal to wider groups, or because of personal prejudices on the part of the authors, would be excluded.

The third step was to determine the content of these topics and their development in each decade. The exercises, questions, and examples in all the books were used for this

---

[2] State and city courses of study, college entrance requirements, periodicals, and books published during the period in question, were examined for lists of books. In addition, Dr. Lao G. Simons' bibliography, a list of American algebra textbooks to 1850, found in the important public and private libraries of the East, was used for the first part of the period. Numerous public and private libraries in the East were consulted.

[3] All parts of the country are represented.

[4] All books were put on an equal basis as it proved impossible to weigh them reliably according to relative influence.

purpose. These were tabulated[5] according to (a) number, (b) specific topic, (c) complexity, and (d) place of appearance, as these points seemed in varying degrees[6] to show the development of different phases of the general topics. In addition the exercises were classified according to type of exercise—briefly, "tool" exercises, "verbal" exercises, and exercises on "theory". All review, supplementary, optional, and test exercises were omitted, in order to make the data from the different books more comparable. The total number of exercises amounted to approximately 500,000. The results were then reduced to the arithmetic means and percentages appearing in the tables.

Naturally the vast number of exercises necessitated classification into relatively large groups which precluded the possibility of fine distinctions. For classification under (b) *specific topic*[7], the exercises were divided into the larger specific topics found under each general topic. In (c) *complexity*[8], two degrees of complexity were used. The first group included the comparatively simple exercises of the topic; the second group, the more complicated exercises. These two groups conform in the main to the classifications made by the National Committee on Mathematical Requirements in its report, *The Reorganization of Mathematics in Secondary Education*, and to the revised requirements of the College Entrance Examination Board. There were, doubtless, cases where exercises that fell in the first group were more complicated than some in the second group. It is believed, however, that the number of such cases is comparatively small, and that the two groups in general represent different levels of complexity. Classification under (d) *place of appearance*, was made as follows: each book was divided by pages into five approximately equal parts and the exercises in each fifth were grouped together. The totals of the exercises in each fifth in the different books were then combined and

---

[5] Each exercise was classified under one general topic with the exception of exercises in general equations and formulas. These were classified twice—once under the latter heading and once under the topic with which they dealt.

[6] In varying degrees only, as each is a function of other variables as well.

[7] Exercises on theory, due to their character, were not classified according to specific topic or complexity.

[8] *Ibid.*

reduced to percentages to obtain the relative distribution by fifths for the decade.

The fourth and final step was the comparison of the tendencies disclosed by these tabulations. Summaries on the various topics have been given at the end of each chapter. A résumé of the central tendencies of the past and present, with a comparison of the present changes in the light of the past, comprises the last chapter. A comparison of the findings with the report of the National Committee on Mathematical Requirements, entitled the *Reorganization of Mathematics in Secondary Education*, is also given in this chapter.

Remarks

1. All conclusions and interpretations have been based on the tables given and hence hold only for the data in these tables. The books used, however, probably represent a large sampling of the books on elementary algebra published during the period. Consequently, the more pronounced or consistent tendencies may be assumed to be indicative of the tendencies in algebra books as a whole.

The sequence of the topics has been based on the relative stress given them in the first decade (1820–29), and not on a psychological or other order found in past or present books. The conclusions for each topic have been given in discussion form at the end of the respective chapters, following the tables and charts on which they are based.

2. Certain abbreviations have been used for the sake of conciseness. "H" and "L" stand respectively for "highest" and "lowest". "Mean number of exercises" refers to the arithmetic mean. The first year of a decade has been used to signify the whole decade, e.g., 1910 indicates the years from 1910–19. In the case of 1920, the division includes the years from 1920 to 1928 inclusive. A few books registered in 1818 and in 1819 have been included with the books of the first decade, 1820 or 1820–29. When reference has been made to the earlier decades of the nineteenth century, the decades from 1820 to 1859 inclusive are meant.

I

Exercises

The aim of this chapter is to give a general view of the development of exercises in elementary algebra from approximately 1820 to the present, irrespective of the topical category under which the exercises appeared. For this purpose all the exercises, with the exception of optional, supplementary, review, and test exercises have been organized into the tables given below. Central tendencies, past and present, that are brought out by these tables are discussed in the latter part of the chapter.

Table 1 shows the mean number of exercises for each decade together with the increase from decade to decade. In Table 2 is given the distribution of these exercises in the books, according to the fifth in which they appeared.

Tables 3 to 11 deal with the different types of exercises—their number, distribution, and relative importance as judged by the emphasis placed on them. The types into which the exercises have been divided are as follows: (a) exercises already expressed in algebraic symbols; (b) exercises requiring translation from English into algebraic symbols, or vice versa; (c) exercises on the theory or development of topics, *i.e.*, questions on "how", "why", etc. For the sake of conciseness, these three groups are respectively referred to in the tables and discussions as *tool* exercises, *verbal* exercises, and exercises on *theory*.

TABLE 1

MEAN NUMBER OF EXERCISES PER BOOK FOR EACH DECADE, TOGETHER WITH A COMPARISON OF THE INCREASE FROM DECADE TO DECADE

DECADE	MEAN NUMBER OF EXERCISES	INCREASE OVER PRECEDING DECADE		
		INCREASE IN NUMBER OF EXERCISES	PER CENT INCREASE	RANK IN PERCENTAGE OF INCREASE
1820	384.5*			
1830	607.9	223.4	58.1	2
1840	856.0	248.1	40.8	3
1850	954.0	98.0*	11.4	8
1860	1509.4	555.4**	58.2**	1**
1870	1673.6	164.2	10.9	9
1880	2184.3	510.7	30.5	4
1890	2620.2	435.9	20.0	5
1900	2740.7	120.5	4.6*	10*
1910	3133.3	392.6	14.6	6
1920	3567.8**	434.5	13.9	7

*Minimum or lowest for the entire period.
**Maximum or highest for the entire period.

Read table as follows: In the decade from 1830 to 1839, the mean number of exercises amounted to 607.9 exercises per book, an increase of 223.4 exercises or a gain of 58.1 per cent over the preceding decade, namely 1820 to 1829. This decade (1830–39) ranked second in the percentage of increase in the mean number of exercises.

TABLE 2

DISTRIBUTION OF EXERCISES ACCORDING TO THE FIFTH IN WHICH THEY APPEARED IN THEIR RESPECTIVE BOOKS

DECADE	PER CENT OF EXERCISES IN THE				
	FIRST FIFTH	SECOND FIFTH	THIRD FIFTH	FOURTH FIFTH	FIFTH FIFTH
1820	27.75**	22.34	20.88	13.65*	15.37
1830	28.10**	20.91	18.18	14.90*	17.91
1840	27.39**	20.69	19.84	16.68	15.39*
1850	25.16**	23.93	17.95	18.05	14.92*
1860	24.24**	22.73	17.31	19.39	16.33*
1870	25.68**	21.85	18.02	17.74	16.70*
1880	23.87**	23.29	17.77	18.90	16.18*
1890	21.45	22.74**	17.55*	20.42	17.84
1900	20.82	23.57**	17.57*	20.43	17.62
1910	22.06	23.02**	21.08	16.88*	16.96
1920	19.62	24.17**	20.82	17.78	17.61*

*Minimum percentage for the decade.
**Maximum percentage for the decade.

Read table as follows: In the first decade, 27.75 per cent of all the exercises for the decade appeared in the first fifth of their respective books; 22.34 per cent appeared in the second fifth; 20.88 per cent, in the third fifth; 13.65 per cent, in the fourth fifth; and 15.37 per cent, in the last fifth.

The percentages in this table have been computed according to the method described on page ix–x.

TABLE 3

MEAN NUMBER OF TOOL EXERCISES, VERBAL EXERCISES, AND EXERCISES ON THEORY IN EACH DECADE

DECADE	TOOL EXERCISES			VERBAL EXERCISES			EXERCISES IN THEORY		
	MEAN NUMBER	CHANGE IN NUMBER OF EXERCISES	PER CENT CHANGE	MEAN NUMBER	CHANGE IN NUMBER OF EXERCISES	PER CENT CHANGE	MEAN NUMBER	CHANGE IN NUMBER OF EXERCISES	PER CENT CHANGE
1820	303.1*			81.4*			00.0*		
1830	394.3	+ 91.2	+30.1	145.5	+ 64.1	+78.7	68.1	+ 68.1	
1840	551.3	+157.0	+39.8	208.6	+ 63.1	+43.4	96.1	+ 28.0	+ 41.1
1850	630.4	+ 79.1	+14.3	211.0	+ 2.4	+ 1.2	112.6	+ 16.5	+ 17.2
1860	915.2	+284.8	+45.2	238.0	+ 27.0	+12.8	356.2**	+243.6	+216.3
1870	1097.5	+182.3	+17.2	287.6	+ 49.6	+20.8	288.5	− 67.7	− 19.0
1880	1698.7	+601.2	+58.4	330.9	+ 43.3	+15.1	154.7	−133.8	− 46.4
1890	2166.8	+468.1	+27.6	334.8	+ 3.9	+ 1.2	118.6	− 36.1	− 23.3
1900	2242.3	+ 75.5	+ 3.5	352.8	+ 18.0	+ 5.4	145.6	+ 27.0	+ 22.8
1910	2424.3	+182.0	+ 8.1	565.6	+212.8	+60.3	143.4	− 2.2	− 1.5
1920	2667.9**	+243.6	+10.0	742.5**	+176.9	+31.3	157.3	+ 13.9	+ 9.7

*Minimum for the entire period.

**Maximum for the entire period.

Read table as follows: In the decade beginning in 1830, the mean number of tool exercises amounted to 394.3 exercises per book, an increase of 91.2 exercises or 30.1 per cent over the decade, 1820–29. The verbal exercises totalled 145.5 in mean number, an increase of 64.1 exercises or 78.7 per cent over the preceding decade. The mean number of exercises on theory in the thirties amounted to 68.1 exercises. This was the first decade in which exercises of this type appeared.

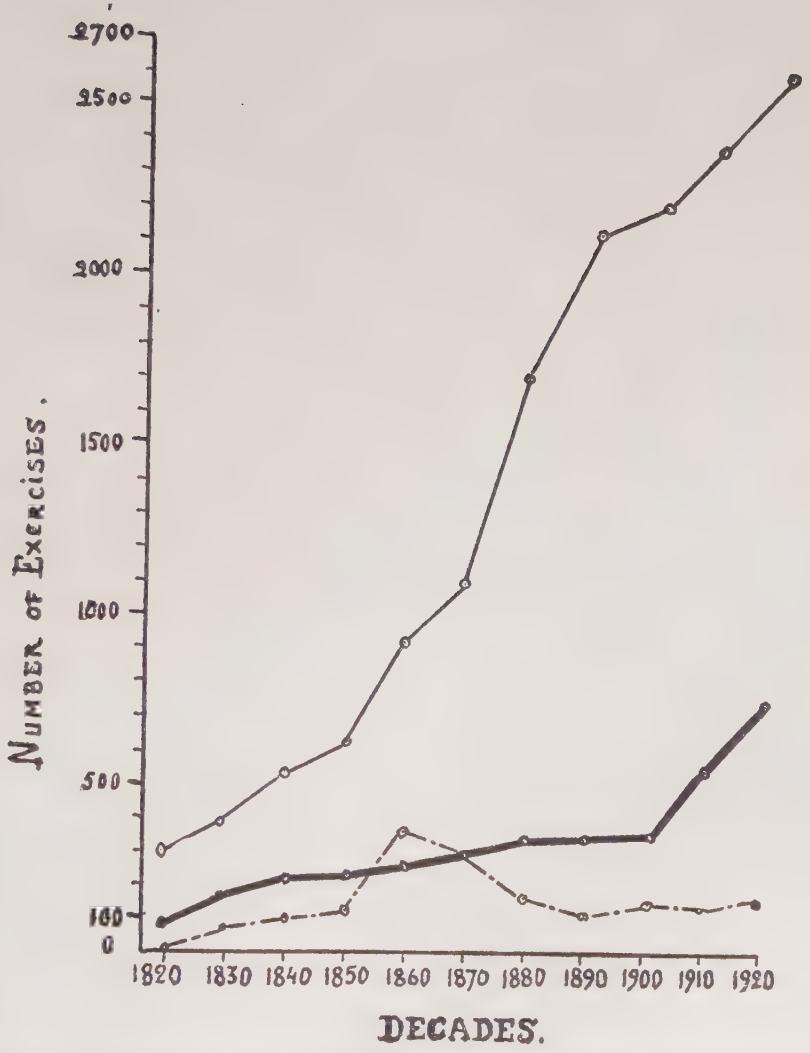


FIG. 1—COMPARISON OF THE MEAN NUMBER OF TOOL EXERCISES, VERBAL EXERCISES, AND EXERCISES ON THEORY

——— Represents mean number of tool exercises.
——— Represents mean number of verbal exercises.
— . — Represents mean number of exercises on theory.

TABLE 4

PERCENTAGE OF TOOL EXERCISES, VERBAL EXERCISES, AND EXERCISES ON THEORY IN EACH DECADE

DECADE	TOOL EXERCISES		VERBAL EXERCISES		EXERCISES ON THEORY	
	PER CENT	CHANGE FROM PRECEDING DECADE	PER CENT	CHANGE FROM PRECEDING DECADE	PER CENT	CHANGE FROM PRECEDING DECADE
1820	78.84		21.17		00.00*	
1830	64.86	−13.98	23.93	+ 2.76	11.21	+11.21
1840	64.40	− .46	24.37**	+ .44	11.23	+ .02
1850	66.08	+ 1.68	22.12	− 2.25	11.81	+ .58
1860	60.62*	− 5.46	15.76	− 6.36	23.61**	+11.80
1870	65.58	+ 4.96	17.18	+ 1.42	17.24	− 6.37
1880	77.77	+12.19	15.15	− 2.03	7.08	−10.16
1890	82.69**	+ 4.92	12.78*	− 2.37	4.53	− 2.55
1900	81.81	− .88	12.87	+ .09	5.31	+ .78
1910	77.37	− 4.44	18.05	+ 5.18	4.58	− .73
1920	74.78	− 2.59	20.81	+ 2.76	4.41	− .17

*Minimum for the entire period.
**Maximum for the entire period.

Read table as follows: In the decade beginning in 1860, tool exercises amounted to 60.62 per cent of all the exercises for the decade. This was a decrease of 5.46 points from the percentage for the decade, 1850–59. The verbal exercises amounted to 15.76 per cent, a decrease of 6.36 points from the preceding decade. Exercises on theory amounted to 23.61 per cent, an increase of 11.80 points over the decade of the fifties. See Fig. 2.

TABLE 5

PERCENTAGE OF (1) VERBAL EXERCISES AND (2) THE OTHER TWO GROUPS COMBINED

DECADE	PER CENT OF	
	VERBAL EXERCISES	EXERCISES ON THEORY AND TOOL EXERCISES
1820	21.17	78.84
1830	23.93	76.07
1840	24.37**	75.63*
1850	22.12	77.89
1860	15.76	84.23
1870	17.18	82.82
1880	15.15	84.85
1890	12.78*	87.22**
1900	12.87	87.13
1910	18.05	81.95
1920	20.81	79.19

*Minimum percentage for the entire period.
**Maximum percentage for the entire period.

Read table as follows: In the first decade, 21.17 per cent of all the exercises were verbal exercises. The other exercises amounted to 78.84 per cent.

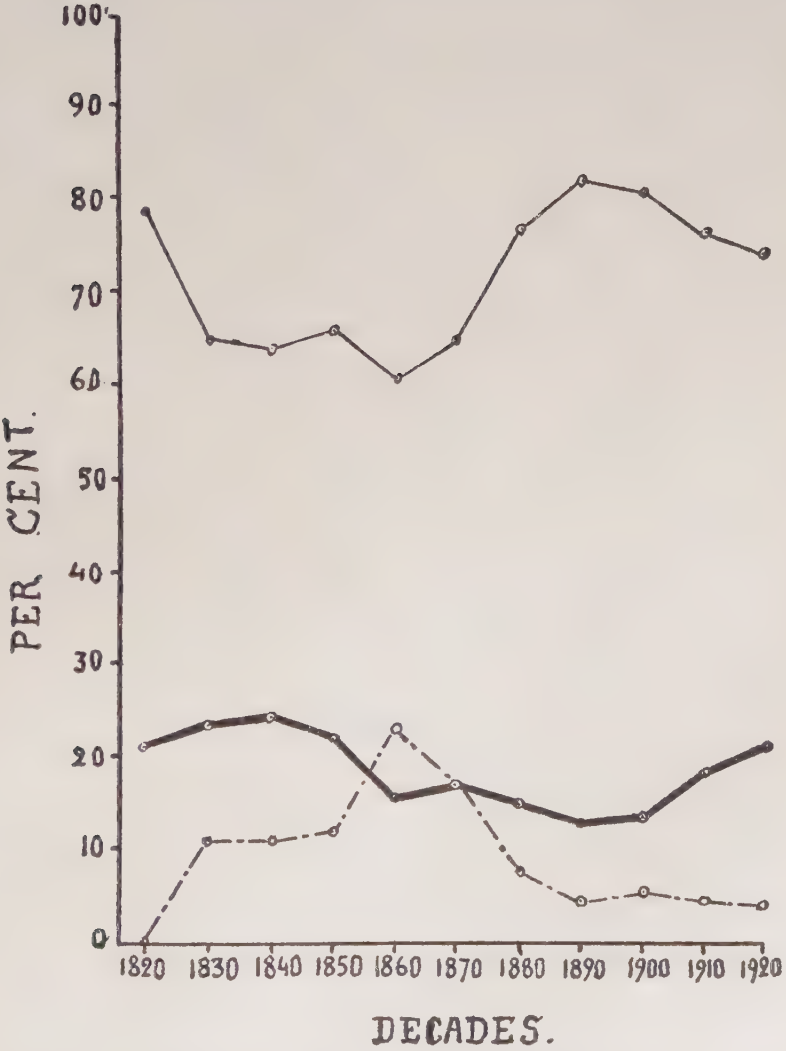


FIG. 2—COMPARISON OF THE PERCENTAGE OF TOOL EXERCISES, VERBAL EXERCISES, AND EXERCISES ON THEORY

_____ Represents the percentage of tool exercises.
_____ Represents the percentage of verbal exercises.
_ . _ Represents the percentage of exercises on theory.

TABLE 6

DISTRIBUTION OF TOOL EXERCISES ACCORDING TO THE FIFTH IN WHICH THEY APPEARED IN THEIR RESPECTIVE BOOKS

DECADE	PER CENT OF THE TOOL EXERCISES IN THE				
	FIRST FIFTH	SECOND FIFTH	THIRD FIFTH	FOURTH FIFTH	FIFTH FIFTH
1820	34.94**	23.95	22.80	10.13	8.18*
1830	34.09**	26.96	14.24	10.87*	13.84
1840	26.43**	24.76	20.17	17.02	11.61*
1850	27.74	29.44**	13.85	18.47	10.50*
1860	22.44	26.73**	14.40*	21.94	14.49
1870	26.00**	24.95	16.02	18.39	14.65*
1880	22.59	26.39**	15.52	20.72	14.78*
1890	19.74	24.20**	16.79*	21.84	17.44
1900	19.30	25.08**	16.20*	21.72	17.70
1910	18.80	23.99**	23.10	16.85*	17.25
1920	16.51	25.11**	22.25	17.54*	18.58

*Minimum percentage for the decade.
**Maximum percentage for the decade.

Read table as follows: In the first decade, 34.94 per cent of the tool exercises appeared in the first fifth of their respective books; 23.95 per cent appeared in the second fifth; 22.80 per cent appeared in the third fifth; 10.13 per cent appeared in the fourth fifth; and 8.18 per cent appeared in the last fifth.

The percentages given in this table are figured according to the method described on page ix–x.

TABLE 7

DISTRIBUTION OF VERBAL EXERCISES ACCORDING TO THE FIFTH IN WHICH THEY APPEARED IN THEIR RESPECTIVE BOOKS

DECADE	PER CENT OF VERBAL EXERCISES IN THE				
	FIRST FIFTH	SECOND FIFTH	THIRD FIFTH	FOURTH FIFTH	FIFTH FIFTH
1820	.98*	16.34	13.76	26.78	42.14**
1830	7.31*	8.74	29.16**	25.77	29.01
1840	29.39**	11.84*	19.99	15.56	23.23
1850	15.21	8.21*	31.06	18.15	27.37**
1860	26.94	8.54*	27.08**	12.37	25.07
1870	21.17	6.96*	28.29	15.26	28.31**
1880	27.38	6.79*	29.92**	11.18	24.72
1890	28.32**	13.92	23.23	13.03*	21.50
1900	26.13	14.37	26.81**	14.02*	18.67
1910	32.40**	20.43	13.33*	16.71	17.13
1920	29.08**	21.42	16.21	18.50	14.78*

*Minimum percentage for the decade.
**Maximum percentage for the decade.

Read table as follows: In the first decade, .98 per cent of the verbal exercises appeared in the first fifth of their respective books; 16.34 per cent appeared in the second fifth; 13.76 per cent, in the third fifth; 26.78 per cent in the fourth fifth; and 42.14 per cent in the last fifth.

The percentages given in this table have been computed according to the method described on page ix–x.

TABLE 8

DISTRIBUTION OF EXERCISES ON THEORY ACCORDING TO THE FIFTH IN WHICH THEY APPEARED IN THEIR RESPECTIVE BOOKS

DECADE	PER CENT OF EXERCISES IN THEORY IN THE				
	FIRST FIFTH	SECOND FIFTH	THIRD FIFTH	FOURTH FIFTH	FIFTH FIFTH
1820	00.00	00.00	00.00	00.00	00.00
1830	37.84**	11.95*	17.51	14.99	17.71
1840	28.56**	16.54*	17.64	17.17	20.08
1850	29.37**	22.53	16.28	15.46*	16.35
1860	27.06**	21.95	18.27	17.52	15.21*
1870	28.99**	24.91	15.37	17.78	12.95*
1880	30.51**	24.53	16.42	15.36	13.19*
1890	33.36**	21.07	15.50	15.28	14.79*
1900	31.45**	22.54	16.31	16.02	13.67*
1910	36.52**	16.72	17.33	18.05	11.39*
1920	27.79**	21.09	18.27	18.36	14.48*

*Minimum percentage for the decade.
**Maximum percentage for the decade.

Read table as follows: In the first decade there were no exercises on theory. In the next decade, 37.84 per cent of the exercises on theory appeared in the first fifth; 11.95 per cent appeared in the second fifth; 17.51 per cent in the third fifth; 14.99 per cent, in the fourth fifth; and 17.71 per cent, in the last fifth.

The percentages given in this table have been computed according to the method described in the statement on page ix–x.

TABLE 9

PERCENTAGE OF TOTAL EXERCISES IN EACH FIFTH THAT WERE TOOL EXERCISES

DECADE	PER CENT THAT WERE TOOL EXERCISES IN THE					NUMBER OF FIFTHS IN WHICH TOOL EXERCISES RANGED FROM		
	FIRST FIFTH	SECOND FIFTH	THIRD FIFTH	FOURTH FIFTH	FIFTH FIFTH	70.1% to 100%	50.1% to 70%	40.1% to 50%
1820	99.25**	84.52	86.05	58.48	41.96*	3	1	1
1830	78.68	83.59**	50.81	47.32*	50.13	2	2	1
1840	62.15	77.07**	65.47	65.72	48.58*	1	3	1
1850	72.85	81.30**	51.01	67.64	46.49*	2	2	1
1860	56.12	71.29**	50.42*	68.60	53.81	1	4	0
1870	66.38	74.88**	58.29	67.96	57.51*	1	4	0
1880	73.57	88.13**	67.94*	85.28	71.08	4	1	0
1890	76.09*	87.98	79.09	88.46**	80.84	5	0	0
1900	75.82	87.07**	75.43*	87.00	82.23	5	0	0
1910	65.92*	80.65	84.82**	77.24	78.70	4	1	0
1920	62.92*	77.70	79.92**	73.78	78.90	4	1	0

*Minimum percentage for the decade.
**Maximum percentage for the decade.

Read table as follows: In the decade, 1870–9, 66.38 per cent of all the exercises in the first fifth were tool exercises. In the second fifth the percentage of tool exercises amounted to 74.88; in the third fifth, to 58.29; in the fourth fifth, to 67.96; and in the last fifth, to 57.51. Thus, in this decade tool exercises composed more than half the exercises in each fifth, in one case exceeding 70 per cent.

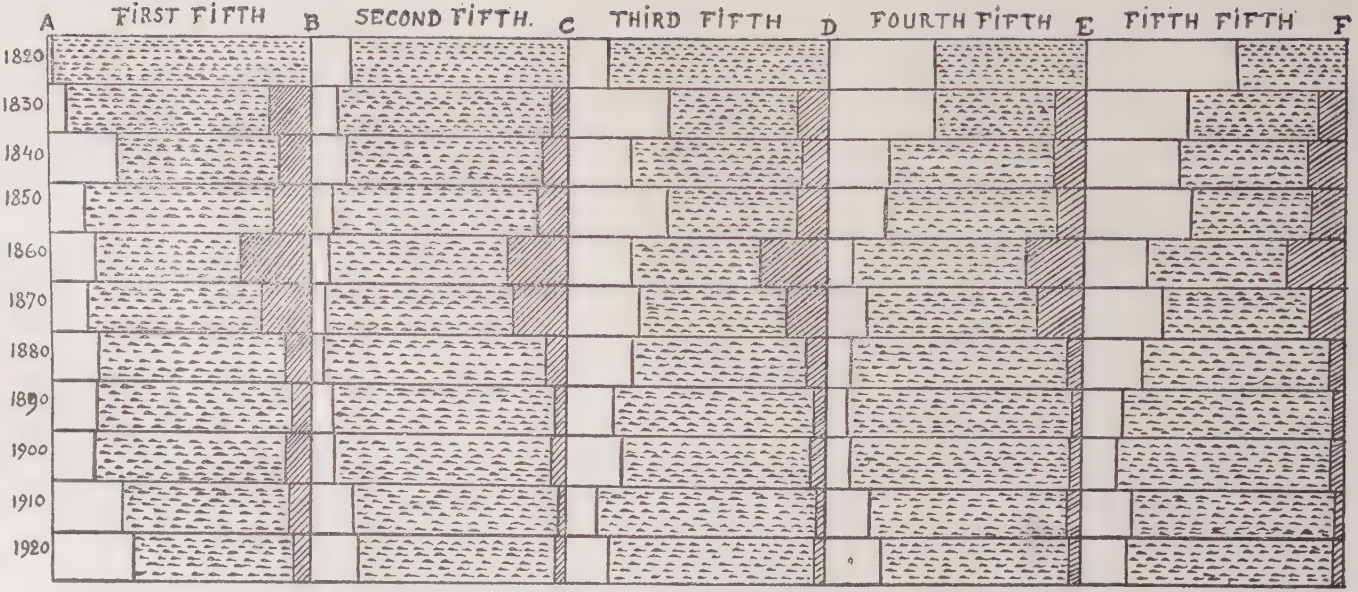
TABLE 10

PERCENTAGE OF TOTAL EXERCISES IN EACH FIFTH THAT WERE VERBAL EXERCISES

DECADE	PER CENT THAT WERE VERBAL EXERCISES IN THE					NUMBER OF FIFTHS IN WHICH VERBAL EXERCISES RANGED FROM			
	FIRST FIFTH	SECOND FIFTH	THIRD FIFTH	FOURTH FIFTH	FIFTH FIFTH	50.1% to 70%	30.1% to 50%	10.1% to 30%	1% to 10%
1820	.75*	15.48	13.95	41.52	58.04**	1	1	2	1
1830	6.23*	10.00	38.39	41.40**	38.78	0	3	0	2
1840	26.15	13.95*	24.55	22.72	36.77**	0	1	4	0
1850	13.37	7.59*	38.28	22.24	40.57**	0	2	2	1
1860	17.52	5.92*	24.66**	10.06	24.20	0	0	4	1
1870	14.17	5.47*	27.00	14.78	29.12**	0	0	4	1
1880	17.38	4.41*	25.51**	8.96	23.15	0	0	3	2
1890	16.87	7.82*	16.91**	8.16	15.40	0	0	3	2
1900	16.15	7.85*	19.64**	8.84	13.65	0	0	3	2
1910	26.51**	16.02	11.42*	17.87	18.23	0	0	5	0
1920	30.84**	18.45	16.21*	21.66	17.47	0	1	4	0

*Minimum percentage for the decade.
**Maximum percentage for the decade.

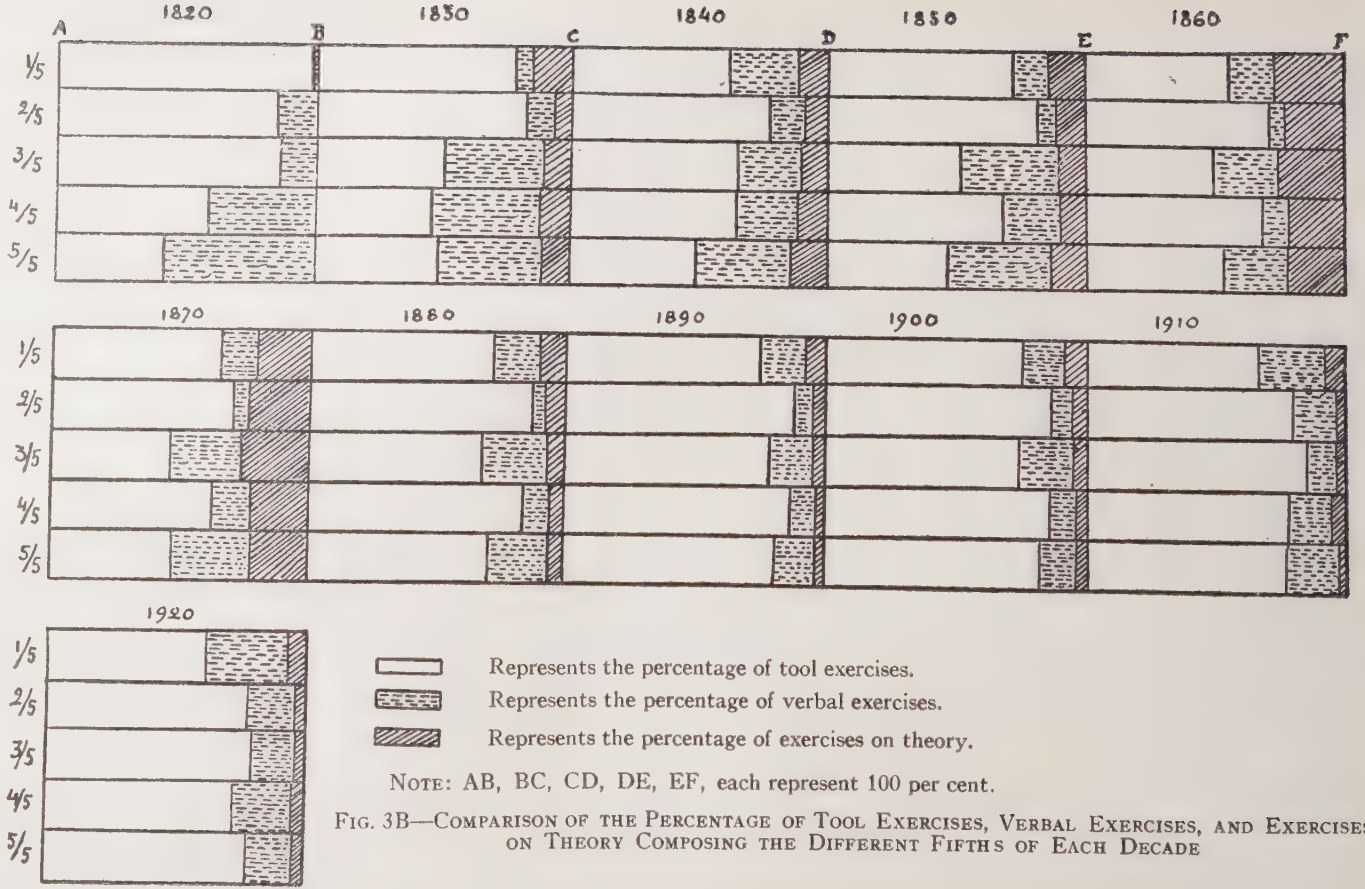
Read table as follows: In the decade, 1890–9, 16.87 per cent of all the exercises in the first fifth were verbal exercises. In the second fifth, the percentage of verbal exercises amounted to 7.82; in the third fifth, to 16.91; in the fourth fifth, to 8.16; and in the last fifth, to 15.40. Thus, in this decade verbal exercises composed less than 30.1 per cent of the exercises in each of the fifths, in two cases being less than 10.1 per cent.



Represents the percentage of verbal exercises.
Represents the percentage of tool exercises.
Represents the percentage of exercises on theory.

NOTE: AB, BC, CD, DE, EF, each represent 100 per cent.

FIG. 3A—COMPARISON OF THE DIFFERENT DECADES IN RESPECT TO THE PERCENTAGE OF TOOL EXERCISES, VERBAL EXERCISES, AND EXERCISES ON THEORY COMPOSING EACH FIFTH



NOTE: AB, BC, CD, DE, EF, each represent 100 per cent.

FIG. 3B—COMPARISON OF THE PERCENTAGE OF TOOL EXERCISES, VERBAL EXERCISES, AND EXERCISES ON THEORY COMPOSING THE DIFFERENT FIFTHS OF EACH DECADE

TABLE 11

PERCENTAGE OF TOTAL EXERCISES IN EACH FIFTH THAT WERE EXERCISES ON THEORY

DECADE	PER CENT THAT WERE EXERCISES ON THEORY IN THE					NUMBER OF FIFTHS IN WHICH EXERCISES IN THEORY RANGED FROM		
	FIRST FIFTH	SECOND FIFTH	THIRD FIFTH	FOURTH FIFTH	FIFTH FIFTH	20.1% to 30%	10.1% to 20%	1.0% to 10%
1820	....	....	....	....	....	....	....	....
1830	15.09**	6.40*	10.80	11.28	11.09	0	4	1
1840	11.71	8.98*	9.98	11.55	14.65**	0	3	2
1850	13.78**	11.11	10.71	10.12*	12.94	0	5	0
1860	26.36**	22.79	24.92	21.34*	21.99	5	0	0
1870	19.46	19.66**	14.71	17.27	13.36*	0	5	0
1880	9.05**	7.46	6.55	5.75*	5.78	0	0	5
1890	7.04**	4.20	4.00	3.39*	3.75	0	0	5
1900	8.03**	5.08	4.93	4.17	4.12*	0	0	5
1910	7.57**	3.32	3.76	4.89	3.07*	0	0	5
1920	6.24**	3.85	3.87	4.56	3.62*	0	0	5

*Minimum percentage for the decade.
**Maximum percentage for the decade.

Read table as follows: In the decade, 1880–9, 9.05 per cent of all the exercises in the first fifth were exercises on theory. In the second fifth, the percentage of exercises on theory amounted to 7.46; in the third fifth, to 6.55; in the fourth fifth, to 5.75; in the last fifth, to 5.78. Thus, in this decade exercises on theory composed less than 10.1 per cent of the exercises in each of the fifths.

Discussion

Table 1 shows that there has been an increase in the mean number of exercises in every decade since the beginning of the period, the whole increasing from 384.5 exercises in the first decade to 3567.8 in the last. The increase from decade to decade varied between 98.0 and 555.4 exercises, the largest increases being in 1860–69 and in 1880–89 and amounting to over 500 exercises in each case. The smallest increase occurred in the fifties and the next smallest in the first decade of the twentieth century. These amounted to 98.0 and 120.5 exercises respectively.

This table shows that the percentages of increase were greatest in the decades of the thirties and the sixties, in each case the increase over the preceding decade being approximately 58 per cent. Three other decades—1840–49, 1880–89, and 1890–99—each showed an increase of 20 per cent or more. The other increases varied between 4.6 and 14.6 per cent, the smallest being in the decade 1900–9.

As is shown in Table 2, the tendency throughout the period has been to have larger percentages of exercises in the first two-fifths of the textbooks. This tendency, however, is not so pronounced now as in the past. The largest percentage appeared in the first fifth in the decades between 1820 and 1880. Since the nineties it has been in the second fifth.

Table 3 and Figure 1 show increases in the mean number of both tool and verbal exercises from the beginning of the period. The increases from decade to decade, however, have generally been much greater in the case of the former. In fact, in only one instance has the increase in verbal exercises exceeded the increase in tool exercises. This was in the decade from 1910 to 1919 when verbal exercises increased in mean number 212.8, exercises and tool exercises only 182.0 In the present decade the verbal exercises increased 176.9 exercises over the decade, 1910–19, and tool exercises, 243.6.

Although the increase in mean number has been greatest in tool exercises in every decade but one, the percentage of increase from decade to decade has frequently been less than that in either verbal exercises or exercises on theory. Only in the decade

of the eighties and in that of the nineties have the percentages of increase in tool exercises been unsurpassed.

In the first decade of the period the mean number of tool exercises amounted to 303.1 exercises. By 1840–49 it had exceeded 500 exercises; by 1870–79, 1000; and by 1890–99, 2000. The mean number of tool exercises in the present decade amounts to 2667.9 exercises.

Verbal exercises amounted to 81.4 in mean number in the first decade. Since then they have increased to 742.5 exercises. The largest percentage of increase was found in the decade of the thirties; the next largest in that beginning in 1910. The former totalled 78.7 per cent and the latter, 60.3 per cent. The smallest increases occurred in the decades beginning in 1850, 1890, and 1900. These amounted to 1.2, 1.2, and 5.4 per cent respectively. The increase of the present decade over that from 1910 to 1919 is 31.3 per cent.

Exercises on theory increased in mean number, as Table 3 shows, from 68.1 in the decade of the thirties, in which they first appeared, to 356.2 in the sixties, in which decade they exceeded the number of verbal exercises. The largest increase took place in that decade, the mean number increasing 243.6 exercises or 216.3 per cent over the preceding decade. The number decreased in the following decade (1870–79) but still surpassed the number of verbal exercises. The tendency to decrease continued to the end of the century, in the last decade of which the mean number amounted to 118.6 exercises. In the present century, the number has varied between 143.4 and 157.3 exercises, with apparently no tendency towards increase or decrease.

Table 4 shows the relative emphasis placed on these three types of exercises in each decade. It is seen that there have been three more or less well defined trends in the stress received by each type. Tool exercises decreased from 78.84 per cent in the first decade to 60.62 per cent in the sixties, a drop of 18.22 points in approximately fifty years. In this interval the decade of the fifties was the only one showing an increase. The greatest decrease occurred in the thirties. An opposite tendency existed from the seventies to the end of the century, the percentage

increasing in each decade until the maximum was reached in the nineties. The largest increase took place in the eighties, the percentage being 12.19 points higher than in the seventies. The present century has shown a return to the earlier tendency, the percentage dropping to 74.78 in the present decade.

Table 4 and Figure 2 show: (1) that the trends in verbal exercises on the whole have been quite the opposite of those in tool exercises; (2) that in some instances the length of the period over which the tendencies persisted has been slightly different. From the beginning of the period studied to the decade, 1840–49, inclusive, verbal exercises increased from 21.17 per cent to 24.37 per cent. The latter percentage is the maximum for the entire period. From the fifties to the end of the century the percentage decreased with the exception of the decade from 1870 to 1879. The minimum was reached in the decade of the nineties and totalled 12.78 per cent, nearly half the percentage in the forties. Verbal exercises have increased in the twentieth century and in the present decade amount to 20.81 per cent. The increase is approximately the same as the decrease in the percentage of tool exercises in this century. On the other hand, the increase in the early part of the nineteenth century and the subsequent decrease were both considerably smaller than the corresponding changes in tool exercises.

Exercises on theory did not appear until the second decade (1830–39). Their appearance was concurrent with the largest decrease in the percentage of tool exercises in the entire period. In that decade 11.21 per cent of all the exercises were on theory. The percentage increased slightly in the two following decades. In the sixties, however, exercises on theory practically doubled in percentage and totalled 23.61 per cent. The percentage has not been so high since. In fact, the percentage of exercises on theory decreased in every decade with the exception of the decade from 1900 to 1909. In the present decade 4.41 per cent of the exercises are of this type. The present percentage is the lowest since the appearance of exercises on theory. However, the percentages in the present century have changed so slightly from decade to decade, that they contrast decidedly with the greatly decreasing ones in the latter part of the last century.

In conclusion, it may be said that tool exercises have always made a much larger percentage of the total exercises than have the other two groups combined. They have comprised between 60 and 80 per cent of the total. The next group in importance, as judged by the relative emphasis placed on it, has usually been that composed of verbal exercises. The latter have always comprised more than 10 per cent of the total, but never so much as 25 per cent. In the decades of the sixties and the seventies this group was stressed the least of the three. Exercises on theory appeared in the thirties and since then, have comprised from approximately 4 to 24 per cent of the total. With the exception of the two decades in which the percentage of exercises on theory exceeded that of verbal exercises, this type has received relatively less emphasis than either of the other two groups.

Tables 6, 7, and 8 show the distribution of the three classes of exercises according to the fifth in which they appeared in their respective books. As is seen in Table 6, the largest percentages of tool exercises have always been in the first two-fifths of the books. However, the trend has been towards gradually reducing the percentage of tool exercises in the first fifth and towards increasing it in the last fifth. With the exception of the seven ies, the largest percentage of this type of exercise has been in the second fifth instead of the first since the first three decades. Hence, there has been a more balanced distribution of tool exercises in the last few decades.

As is evident from Table 7, the tendency in verbal exercises has been somewhat the opposite of the tendency in the tool exercises; *i.e.*, there has been an increase in the first fifth of the books and a decrease in the last two fifths. Only .98 per cent of the verbal exercises were in the first fifth in the decade beginning in 1820, but in the forties there were 29.39 per cent. This increase has been more or less maintained since then, with the exception of the decade from 1850 to 1859, in which ten-year period the percentage dropped to 15.21. The last fifth has decreased from 42.14 per cent in the first decade to 14.78 per cent in the present decade. The fourth fifth has also shown a decrease. As a result there is also a more even distribution of verbal exercises.

The largest percentage of exercises on theory has been in the first fifth throughout the period in which these exercises have appeared, judging from Table 8. The smallest percentage has usually been in the last fifth of the books. There is no appreciable tendency towards changing the distribution of exercises on theory.

Tables 9, 10, and 11 and Figures 3a and 3b show the composition of the fifths in each decade; *i.e.*, what part of each fifth is composed of tool exercises, verbal exercises, and exercises on theory. Table 9 shows that tool exercises have been in the majority in every fifth of every decade with but four exceptions. These four exceptions were all in the earlier decades. Since the beginning of the second half of the nineteenth century, every fifth has had more than 50 per cent of its exercises belonging to this type. Since and including the decade of the seventies, at least four of the fifths in each decade have had over 70 per cent tool exercises. In the last decade of the nineteenth century and in the first decade of the twentieth century, every fifth was composed of more than 70 per cent tool exercises.

A gradual change in the relative stress laid on this type of exercise is evident from Figure 3a. This is especially noticeable in the first fifths. In the earlier decades of the period the last fifths had smaller percentages of tool exercises than the other parts of the books. In the present decade this is true of the first fifth instead of the last. Furthermore, in the two most recent decades it is the third fifth instead of the second fifth that has the largest percentage of its exercises belonging to this group. Therefore, Figure 3a and Table 9 show, as does Table 6 from a different angle, that the stress on tool exercises is gradually lessening in the earlier fifths.

Table 10 shows the percentage of the total exercises in each fifth that are verbal exercises. It is evident that only the last fifth of the first decade had more than half of its exercises in this group. In a few fifths in the first four decades and in the first fifth of the present decade, verbal exercises have varied between 30.1 and 50 per cent. In the vast majority of the fifths, however, less than 30 per cent of the exercises have been verbal, a number of the fifths being even below 10 per cent.

Figure 3a shows the gradual change that has been taking place

in the relative stress on verbal exercises. In the earlier decades the last fifth generally had a larger percentage of verbal exercises than the other fifths. In the second half of the nineteenth century and in the first decade of the twentieth century, the largest percentage came in the third or middle fifth. In the present decade it is found in the first fifth.

Table 11 shows the percentage of the exercises in each fifth that are on theory. It is seen that in the last fifty years, exercises on theory have comprised less than ten per cent of the exercises in each fifth. The greatest emphasis on exercises of this type was placed on them in the decades between 1830 and 1879. In this interval the percentages of exercises on theory in the different fifths were not only larger than in the other decades, dut at times exceeded the percentages of verbal exercises. This was frequently the case in the first and second fifths. In the decade of the sixties, four of the five fifths had larger percentages of exercises on theory than of verbal exercises. In that decade more than one in every five exercises in each fifth were on theory. Both Figure 3a and Figure 3b show the preponderance of exercises on theory over the verbal exercises in the middle decades of the last century.

II

Equations

In the first part of this chapter are given the tables and graphs illustrating the development of equations by decades from the beginning of the period examined to the present. These are then discussed in the latter part of the chapter.

In Table 12 is shown the mean number of exercises, together with the corresponding increase or decrease in each decade. The percentage of the total number of exercises in each decade that were devoted to equations is given in Table 13. In this table are also given the changes in percentage from the decade immediately preceding and from the first decade. Table 14 indicates the distribution of the exercises in the books, the percentage of exercises that appeared in each fifth being used for this purpose.

Simple and complicated exercises in equations are compared in Table 15. Easy equations of the first and second degrees in one unknown and sets of easy equations of the first degree in two unknowns, make up the simpler group. The more complicated group is composed of: (a) complicated equations of the first and second degrees in one unknown; (b) sets of complicated equations of the first degree in two unknowns; sets of equations of the second or higher degree; sets of equations in three or more unknowns; (c) radical equations; (d) cubic and higher equations, etc.

The stress given to various kinds of equations is shown in Table 16. Here are given the percentages of exercises in equations devoted respectively to (a) linear equations, (b) quadratic equations, (c) simultaneous equations, and (d) radical equations. The fifth percentage comprises the exercises in all other types of equations. In (a), *linear equations*, are included all equations of the first degree in one unknown, and in (b), *quadratic equations*, are all equations of the second degree in one unknown. The third group (c), *simultaneous equations*, comprises all sets of equations in two or more unknowns and the

fourth group (d), *radical equations*, all equations in one unknown in which the latter appears under the radical sign or with a fractional exponent.

TABLE 12

MEAN NUMBER OF EXERCISES IN EQUATIONS PER BOOK, TOGETHER WITH A COMPARISON OF THE INCREASE OR DECREASE IN EACH DECADE

DECADE	MEAN NUMBER OF EXERCISES	CHANGE FROM PRECEDING DECADE		
		NUMBER OF EXERCISES	PER CENT INCREASE OR DECREASE	RANK IN PERCENTAGE OF INCREASE
1820	137.3*			
1830	211.2	+ 73.9	+53.8	2
1840	288.6	+ 77.4	+36.6	3
1850	320.7	+ 32.1	+11.1	8
1860	512.0	+191.3	+59.7	1**
1870	468.6	− 43.4	− 8.5	10*
1880	602.2	+133.6	+28.5	4
1890	691.2	+ 89.0	+14.8	7
1900	758.7	+ 67.5	+ 9.8	9
1910	935.2	+176.5	+23.3	5
1920	1105.8**	+170.6	+18.2	6

*Minimum or lowest for the entire period.
*Maximum or highest for the entire period.

Read table as follows: In the decade beginning in 1830, the mean number of exercises in equations amounted to 211.2 exercises per book, an increase of 73.9 exercises over the decade of the twenties or a gain of 53.8 per cent. This decade ranked second in the percentage of increase in mean number of equations.

TABLE 13

PERCENTAGE OF EXERCISES IN EQUATIONS, TOGETHER WITH A COMPARISON OF THE INCREASE OR DECREASE IN EACH DECADE

DECADE	EXERCISES IN EQUATIONS			
	PER CENT	CHANGE FROM PRECEDING DECADE	CHANGE FROM 1820	RANK IN PERCENTAGE OF EXERCISES
1820	35.71**			1**
1830	34.74	− .97	− .97	2
1840	33.72	−1.02	−1.99	4
1850	33.62	− .10	−2.09	5
1860	33.91	+ .29	−1.80	3
1870	28.00	−5.91	−7.71	8
1880	27.57	− .43	−8.14	10
1890	26.38*	−1.19	−9.33	11*
1900	27.68	+1.30	−8.03	9
1910	29.85	+2.17	−5.86	7
1920	30.99	+1.14	−4.72	6

*Minimum or lowest for the entire period.
**Maximum or highest for the entire period.

Read table as follows: In the decade beginning in 1900, 27.68 per cent of the exercises for the decade were exercises in equations. This was an increase of 1.30 points over the percentage for the decade of the nineties, but a decrease of 8.03 points from the percentage for the decade, 1820–29. This decade (1900–09) ranked ninth in the emphasis placed on equations.

See Fig. 4.

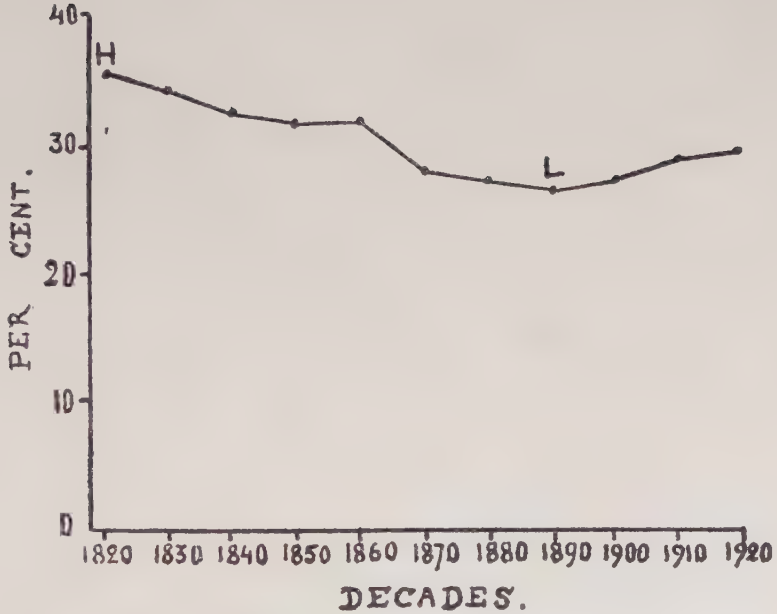


FIG. 4—PERCENTAGE OF EXERCISES IN EQUATIONS

TABLE 14

DISTRIBUTION OF EXERCISES IN EQUATIONS ACCORDING TO THE FIFTH IN WHICH THEY APPEARED IN THEIR RESPECTIVE BOOKS

DECADE	PER CENT OF EQUATIONS IN THE				
	FIRST FIFTH	SECOND FIFTH	THIRD FIFTH	FOURTH FIFTH	FIFTH FIFTH
1820	11.29	17.77	10.85*	29.72	30.37**
1830	6.83*	18.26	25.09	30.37**	19.45
1840	16.93*	17.95	20.76	24.77**	19.59
1850	12.47*	13.71	30.21**	22.97	20.64
1860	17.16	10.90*	23.92	18.21	29.82**
1870	7.34*	15.56	30.73**	23.56	22.81
1880	11.48	8.20*	33.40**	23.43	23.49
1890	12.18	10.77*	25.90	21.33	29.82**
1900	10.31*	12.41	29.18**	22.87	25.22
1910	17.58	17.12*	18.00	22.53	24.77**
1920	21.55	18.78	17.04*	20.59	22.04**

*Minimum percentage for the decade.
**Maximum percentage for the decade.

Read table as follows: In the first decade, 11.29 per cent of the exercises in equations appeared in the first fifth of their respective books; 17.77 per cent appeared in the second fifth; 10.85 per cent, in the third fifth; 29.72 per cent, in the fourth fifth; and 30.37 per cent, in the last fifth.

The percentages given in this table have been computed according to the method described on page ix–x.

TABLE 15

PERCENTAGE, RESPECTIVELY, OF SIMPLE AND COMPLICATED EXERCISES IN EQUATIONS FOR EACH DECADE

DECADE	PER CENT OF EXERCISES	
	SIMPLE	COMPLICATED
1820	69.56	30.45
1830	79.41	20.59
1840	71.04	28.96
1850	70.63	29.37
1860	65.25	34.75
1870	72.49	27.51
1880	62.91*	37.09**
1890	66.27	33.73
1900	69.47	30.53
1910	82.33	17.67
1920	90.20**	9.80*

*Minimum percentage for the entire period.
**Maximum percentage for the entire period.

Read table as follows: In the first decade, 69.56 per cent of the exercises in equations were simple exercises and 30.45 per cent were complicated.

The types of exercises in equations that compose the two groups—simple and complicated—are listed on page 22. See Fig. 5.

TABLE 16

PERCENTAGES OF EXERCISES IN EQUATIONS DEVOTED TO LINEAR, QUADRATIC, SIMULTANEOUS, RADICAL, AND UNCLASSIFIED EQUATIONS, RESPECTIVELY

DECADE	PER CENT OF EQUATIONS DEVOTED TO				
	LINEAR EQUATIONS	QUADRATIC EQUATIONS	SIMULTANEOUS EQUATIONS	RADICAL EQUATIONS	UNCLASSIFIED EQUATIONS
1820	40.57	16.75*	29.21	9.61**	3.86**
1830	48.60	17.48	29.24	3.22	1.46
1840	42.79	18.95	31.00	5.80	1.46
1850	43.72	19.13	31.54	4.37	1.25
1860	41.62	20.94	27.22	7.88	2.33
1870	41.61	19.42	31.69**	6.18	1.11
1880	38.82	19.68	30.69	7.58	3.23
1890	37.54*	22.29	31.16	6.07	2.94
1900	40.21	23.83**	27.25	5.78	2.92
1910	51.37	21.74	21.97	2.72	2.19
1920	63.73**	18.56	15.43*	1.63*	.65*

*Minimum percentage for the entire period.
**Maximum percentage for the entire period.

Read table as follows: In the first decade, 40.57 per cent of the exercises in equations were linear equations; 16.75 per cent were quadratic equations; 29.21 per cent were simultaneous equations; 9.61 per cent were radical equations; and 3.86 per cent were other kinds of equations. See Fig. 6.

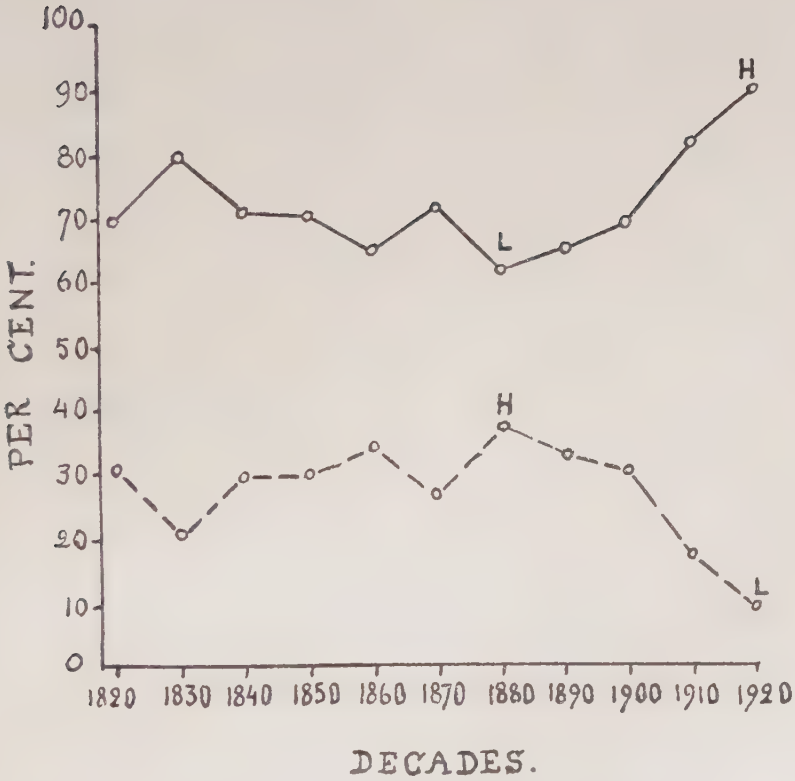


FIG. 5—COMPARISON OF SIMPLE AND COMPLICATED EXERCISES IN EQUATIONS

_____ Represents simple exercises in equations.
_ . _ Represents complicated exercises in equations.

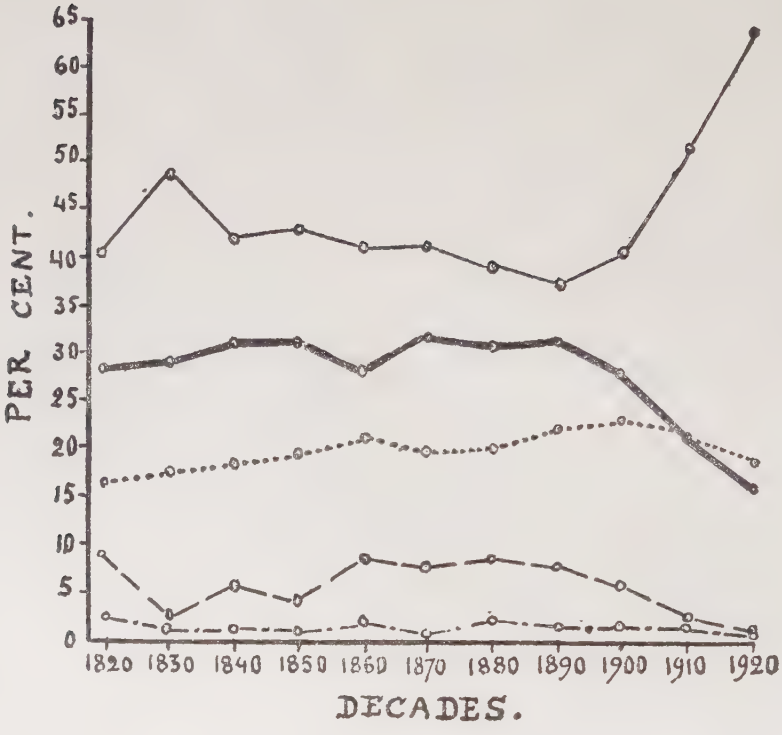


FIG. 6—COMPARISON OF LINEAR, QUADRATIC, SIMULTANEOUS, RADICAL AND UNCLASSIFIED EQUATIONS

——— Represents exercises in linear equations.
. . . . Represents exercises in quadratic equations.
——— Represents exercises in simultaneous equations.
— — — Represents exercises in radical equations.
— . — Represents exercises in unclassified equations.

Discussion

Table 12 shows that the mean number of exercises in equations has generally increased. The minimum, 137.3 exercises, came in the first decade. The maximum, 1105.8 exercises, is found in the present decade. The largest increase took place in the decade beginning in 1860, when the mean number increased 191.3 exercises or 59.7 per cent over 1850–59. In the next decade occurred the only decrease in the entire period. This amounted to 8.5 per cent. In the present decade the increase over 1910–19 amounts to 18.2 per cent or 170.6 exercises.

The data in Table 13 show that equations have ranged from 26.38 per cent to 35.71 per cent of the number of exercises. The general tendency in the nineteenth century was to decrease the percentage; the general tendency in the present century so far has been to increase it. The maximum percentage for the period occurred in the first decade (1820–29) and amounted to 35.71 per cent. The minimum, 26.38 per cent, came in the decade beginning in 1890.

The greatest decrease in the last century took place in the decade of the seventies. From 33.91 per cent in the decade preceding, the percentage dropped in the seventies to 28.00. Previous to that decade, more than a third of all the exercises were in equations. This has not been the case, however, since the sixties, although each of the three decades of the present century shows an increase over the decade immediately preceding it. In the present decade 30.99 per cent of the exercises are equations, an increase of 4.61 points over the last decade of the nineteenth century.

As is indicated in Table 14, more exercises in equations have appeared in the latter parts of the books than in the earlier parts. In fact, over 40 per cent of all equations have been in the last two-fifths throughout the period, whereas with the exception of three decades only between 20 and 30 per cent have been in the first two-fifths combined. By grouping the first two-fifths, it may clearly be seen that the greatest amount of early drill in the last century was given in the decade from 1840 to 1849. This amounted to 34.88 per cent. It was not until the last two decades that early drill in equations was stressed to this extent. In the decade beginning in 1910 the percentage of drill

in the first two-fifths amounted to 34.70 per cent. In the present decade it amounts to 40.33 per cent.

Table 15 shows that simple exercises in equations increased in percentage in the second decade. In that decade (1830–39) they amounted to 79.41 per cent of the total number of equations. However, in the decades following 1830–1839 the percentage decreased with the exception of the seventies. The minimum was reached in the decade of the eighties. This amounted to 62.91 per cent, a decrease of 16.50 points from the percentage in the thirties. Since the decade, 1880–89, however, the percentage of simple exercises has increased. The increase has been especially pronounced in the last two decades, both of which have higher percentages of simple exercises than did the decade from 1830 to 1839. The percentage in the present decade amounts to 90.20.

The proportion of complicated exercises decreased in the thirties, but in the decades from 1840 to 1889 inclusive it generally increased. In the decade of the eighties more than one in every three equations were complicated. The percentage, however, has steadily decreased since then. In the present decade less than one equation in ten is complicated. This is less than in 1830 to 1839, in which decade approximately two equations in every ten were complicated.

In Table 16 are compared the different kinds of equations. Linear equations are shown to have comprised throughout the period more than one-third of all the exercises in equations. In the first decade (1820–29), linear equations amounted to 40.57 per cent. This increased to 48.60 per cent in the next decade. The percentage decreased in the following decades with the exception of the seventies until the minimum, 37.54 per cent, was reached in the decade of the nineties. Linear equations have increased in percentage in the three decades of the present century with the result that now 63.73 per cent of all equations are of this type.

Quadratic equations amounted to 16.75 per cent of all equations in the first decade. They increased slightly in each decade for a period of forty years (1840–1869). In the sixties they amounted to 20.94 per cent and in the seventies to 19.42 per cent. In the following three decades (1880–1909) the tendency

to increase in percentage continued. The maximum, 23.83 per cent, came in the first decade of the twentieth century. In the last two decades (1910–28), quadratic equations have decreased in percentage. In the present decade they amount to 18.56 per cent, approximately the same percentage as in the decade of the forties.

Simultaneous equations were second in importance to linear equations until this decade, when they dropped to 15.43 per cent. This is lower than the percentage of quadratic equations Previous to this decade, simultaneous equations constituted from 21.97 to 31.69 per cent of the exercises in equations.

Radical equations received the most emphasis—9.61 per cent—in the first decade of the period and the least—1.63 per cent—in the last decade. With the exception of the beginning decade, however, the percentages were lower preceding the sixties than they were following it. Radical equations have tended to decrease in emphasis in the last forty years, *i.e.*, since approximately 1890. In the present decade only 1.63 per cent of the equations are of this kind.

All equations not included in the types discussed above, amount to .65 per cent in the present decade. In the other decades they aggregated from 1.11 to 3.86 per cent.

Summary

Linear equations have always constituted the largest percentages of exercises in equations. They have ranged from slightly over one exercise in three to one in two. Simultaneous equations were second in number in ten of the eleven decades of the period, the only exception being the present decade. Except in 1920–28 and 1910–19 more than one in every four equations were of this type. Approximately one in every five or six equations have been of the second degree, and always less than one in ten have been radical equations.

Recently equations have received more stress than they did in the last half of the nineteenth century, but they are still emphasized to a smaller extent than they were in the early decades of the period. In the present decade complicated equations are less numerous, comparatively speaking, than at any other time in the period.

III

Roots and Radicals

The tables and charts in the first part of this chapter show the development of roots and radicals. These developments are discussed at the end of the chapter.

Table 17 gives the mean number of exercises for each decade, together with the percentage of change in the mean number from decade to decade. Table 18 shows the percentage of exercises for each decade, with the corresponding increases or decreases from decade to decade and from 1820. In Table 19 is given the distribution of the exercises in this topic according to the fifth in which they appeared in their respective books.

In Table 20 is given a comparison of the percentage of simple and complicated exercises in roots and radicals. The following were considered simple exercises: square root of numbers; square root of perfect square trinomials, monomial radicals of the first and second degrees, and exercises in the evaluation of simple radical expressions. The complicated group comprises: cube and higher roots of numbers by long division, roots of algebraic expressions by long division, non-monomial radicals above the third degree, fundamental operations with radicals, and radicals with literal and negative exponents.

Table 21 shows the development of different phases of roots and radicals. Here is given the percentage of radicals having fractional exponents, the percentage of exercises in roots of numbers, the percentage in roots of algebraic expressions, and the percentage of all other radicals.

TABLE 17

MEAN NUMBER OF EXERCISES IN ROOTS AND RADICALS PER BOOK, TOGETHER WITH A COMPARISON OF THE INCREASE OR DECREASE IN EACH DECADE

DECADE	MEAN NUMBER OF EXERCISES	CHANGE FROM PRECEDING DECADE		
		NUMBER OF EXERCISES	PER CENT INCREASE OR DECREASE	RANK IN PERCENTAGE OF INCREASE
1820	68.6*			
1830	74.6	+ 6.0	+ 8.7	7
1840	150.4	+ 75.8	+101.6	1**
1850	126.0	− 24.4	− 16.3	9
1860	194.8	+ 68.8	+ 54.6	2
1870	220.0	+ 25.2	+ 12.9	6
1880	313.0	+ 93.0	+ 42.3	3
1890	424.6	+111.6	+ 35.7	4
1900	426.3**	+ 1.7	+ .4	8
1910	294.6	−131.7	− 30.9	10*
1920	334.9	+ 40.3	+ 13.7	5

*Minimum or lowest for the entire period.
**Maximum or highest for the entire period.

Read table as follows: In the decade beginning in 1830, the mean number of exercises in roots and radicals amounted to 74.6 exercises per book, an increase of 6.0 exercises or a gain of 8.7 per cent over the first decade, 1820–29. The decade of the thirties ranked seventh in the percentage of increase in mean number of exercises in roots and radicals.

TABLE 18

PERCENTAGE OF EXERCISES IN ROOTS AND RADICALS, TOGETHER WITH A COMPARISON OF THE INCREASE OR DECREASE IN EACH DECADE

DECADE	EXERCISES IN ROOTS AND RADICALS			
	PER CENT	CHANGE FROM PRECEDING DECADE	CHANGE FROM 1820	RANK IN PERCENTAGE OF EXERCISES
1820	17.84**			1**
1830	12.28	−5.56	−5.56	9
1840	17.57	+5.29	− .27	2
1850	13.21	−4.36	−4.63	6
1860	12.98	− .23	−4.86	8
1870	13.15	+ .17	−4.69	7
1880	14.33	+1.18	−3.51	5
1890	16.20	+1.87	−1.64	3
1900	15.55	− .65	−2.29	4
1910	9.40	−6.15	−8.44	10
1920	9.39*	− .01	−8.45	11*

*Minimum or lowest for the entire period.
**Maximum or highest for the entire period.

Read table as follows: In the decade from 1880 to 1889, exercises in roots and radicals constituted 14.33 per cent of the total exercises for the decade. This was an increase of 1.18 points over the percentage for the decade, 1870–79; but a decrease of 3.51 points from the percentage for the first decade, 1820–29. This decade ranked fifth in the emphasis placed on roots and radicals. See Fig. 7.

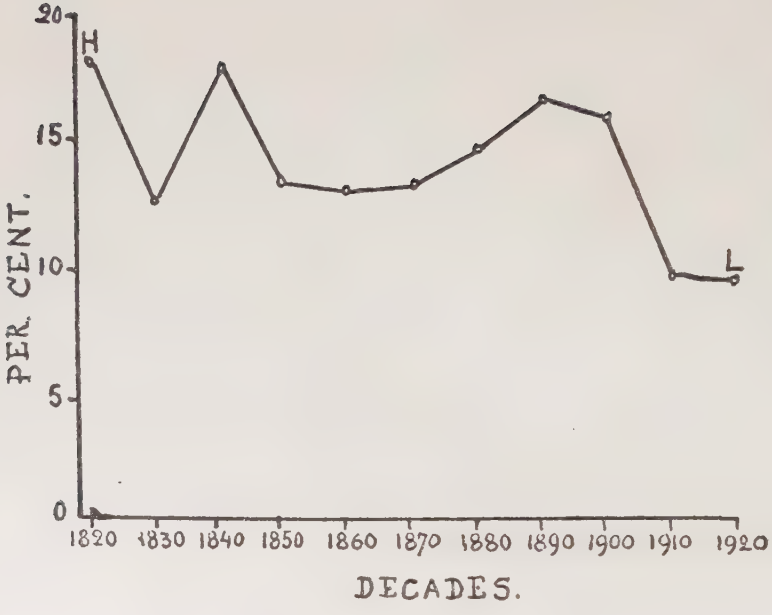


FIG. 7—PERCENTAGE OF EXERCISES IN ROOTS AND RADICALS

TABLE 19

DISTRIBUTION OF EXERCISES IN ROOTS AND RADICALS ACCORDING TO THE FIFTH IN WHICH THEY APPEARED IN THEIR RESPECTIVE BOOKS

DECADE	PER CENT OF ROOTS AND RADICALS IN THE				
	FIRST FIFTH	SECOND FIFTH	THIRD FIFTH	FOURTH FIFTH	FIFTH FIFTH
1820	4.08*	14.14	60.35**	7.29	14.14
1830	2.20*	6.79	34.93**	21.15	34.93**
1840	.77*	10.61	36.45**	35.22	16.96
1850	1.82*	5.55	33.52	46.84**	12.26
1860	.74*	.85	15.14	80.37**	2.89
1870	1.47*	4.28	10.91	64.36**	18.98
1880	.64*	1.47	11.36	69.79**	16.74
1890	.71*	4.33	5.82	64.32**	24.82
1900	.22*	.98	5.39	60.00**	33.41
1910	.43*	4.60	7.56	29.63	57.79**
1920	.59*	1.02	9.90	31.69	56.80**

*Minimum percentage for the decade.
**Maximum percentage for the decade.

Read table as follows: In the first decade, 4.08 per cent of the exercises in roots and radicals appeared in the first fifth of their respective books; 14.14 per cent appeared in the second fifth; 60.35 per cent, in the third fifth; 7.29 per cent, in the fourth fifth; and 14.14 per cent, in the last fifth.

The percentages given in this table have been computed according to the method described on page ix–x.

TABLE 20

PERCENTAGE, RESPECTIVELY, OF SIMPLE AND COMPLICATED EXERCISES IN ROOTS AND RADICALS FOR EACH DECADE

DECADE	PER CENT OF EXERCISES	
	SIMPLE	COMPLICATED
1820	23.76*	76.24**
1830	34.13	65.87
1840	29.05	70.95
1850	36.82	63.18
1860	33.83	66.17
1870	30.95	69.05
1880	28.32	71.68
1890	24.99	75.01
1900	25.16	74.84
1910	44.51	55.49
1920	52.48**	47.52*

*Minimum percentage for the entire period.
**Maximum percentage for the entire period.

Read table as follows: In the first decade, 23.76 per cent of the exercises in roots and radicals were simple exercises, and 76.24 per cent were complicated.

The types of exercises in roots and radicals that compose the two groups—simple and complicated—are listedon page 32. See Fig 8.

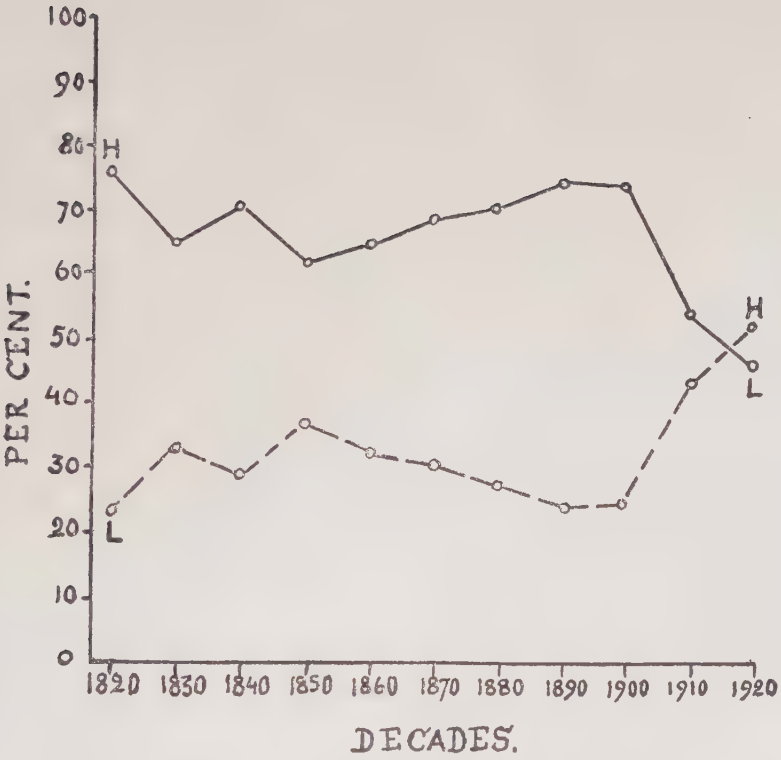


FIG. 8—COMPARISON OF SIMPLE AND COMPLICATED EXERCISES IN ROOTS AND RADICALS

— — — Represents simple exercises in roots and radicals.
——— Represents complicated exercises in roots and radicals.

TABLE 21

PERCENTAGE OF EXERCISES IN ROOTS AND RADICALS DEVOTED TO ROOTS OF ALGEBRAIC EXPRESSIONS, ROOTS OF NUMBERS, RADICALS WITH FRACTIONAL EXPONENTS, AND UNCLASSIFIED RADICALS, RESPECTIVELY

DECADE	PER CENT OF ROOTS AND RADICALS DEVOTED TO			
	ROOTS OF ALGEBRAIC EXPRESSIONS	ROOTS OF NUMBERS	RADICALS WITH FRACTIONAL EXPONENTS	UNCLASSIFIED RADICALS
1820	13.27	8.60	18.08	60.06
1830	12.10	15.88	11.99*	60.04
1840	9.06	18.41	18.16	54.37
1850	11.99	23.73**	14.00	50.27
1860	15.28**	14.32	23.66**	46.74*
1870	7.62	8.56*	19.22	64.60
1880	8.48	13.87	22.17	55.48
1890	8.89	9.45	18.53	63.13
1900	8.48	9.81	17.52	64.19
1910	8.80	14.04	12.15	65.01**
1920	7.46*	13.79	15.31	63.42

*Minimum percentage for the entire period.
**Maximum percentage for the entire period.

Read table as follows: In the first decade, 13.27 per cent of the exercises in roots and radicals were roots of algebraic expressions; 8.60 per cent were roots of numbers; 18.08 per cent were radicals with fractional exponents; and 60.06 per cent were other radicals. See page 9.

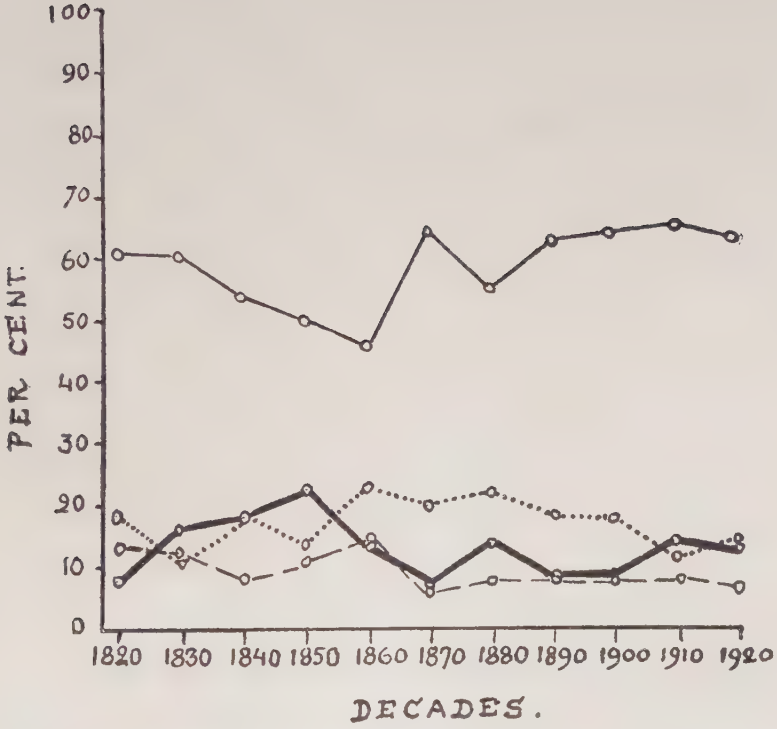


FIG. 9—COMPARISON OF ROOTS OF ALGEBRAIC EXPRESSIONS, ROOTS OF NUMBERS, RADICALS WITH FRACTIONAL EXPONENTS, AND UNCLASSIFIED RADICALS

_ _ _ _ Represents exercises in roots of algebraic expressions.
_____ Represents exercises in roots of numbers.
. . . . Represents exercises in radicals with fractional exponents.
_____ Represents exercises in unclassified radicals.

Discussion

Table 17 shows that the mean number of exercises in roots and radicals increased from 68.6 exercises, the minimum for the period, in the first decade to 426.3 exercises, the maximum, in the decade beginning in 1900. The greatest increase amounted to 101.6 per cent and occurred in the decade of the forties. The only decrease in the last century appeared in the decade from 1850 to 1859 and amounted to 16.3 per cent. In the second decade of the twentieth century, the mean number of exercises decreased nearly one-third from the maximum in 1900–09. In the present decade, the mean number, 334.9 exercises, is higher than it was in 1910–19. It is, however, still approximately 20 per cent lower than in the first decade of the century.

As is indicated in Table 18 and Figure 7, the highest percentage of exercises in roots and radicals amounted to 17.84 and occurred in the first decade of the period. In the decade from 1830 to 1839 the percentage decreased to 12.28; but this decrease was only temporary for in the next decade the exercises in this topic once more nearly equalled 18 per cent. The percentages in the next two decades were appreciably lower, but these were followed by gradually increasing percentages to the end of the century. Since the beginning of the present century, however, the percentages have once more decreased, the largest decrease taking place in the decade from 1910 to 1919. The minimum for the period is found in the present decade and amounts to 9.39 per cent.

The data in Table 19 show that the largest percentage of exercises in roots and radicals has gradually changed from the third to the last fifth of the books. In the decades of the twenties and the forties the largest number was found in the third fifth; in the decade of the thirties an equally large number was found in the last fifth. In the decades from 1850 to 1909 the largest percentage appeared in the fourth fifth. Since then it has been in the last fifth. This tendency towards later drill may be clearly seen by examining the percentages in each fifth for the entire period. The first fifth dropped from 4.08 per cent in the first decade to .59 in the last. In the same decades the second fifth

decreased from 14.14 per cent to 1.02 and the third fifth from 60.35 per cent to 9.90. On the other hand, the fourth fifth during this interval increased from 7.29 per cent in the first decade to 31.69 per cent in the last decade, and the last fifth from 14.14 per cent to 56.80 per cent.

Simple and complicated exercises in roots and radicals are compared in Table 20. From the decade of the twenties to that of the fifties, the number of simple exercises varied between 23.76 per cent and 36.82 per cent. From the decade beginning in 1860 to the end of the century the percentage decreased. The tendency in the present century has been in the other direction, simple exercises in roots and radicals increasing to 52.48 per cent in the present decade. The most pronounced increase took place in the decade from 1910 to 1919.

Preceding the last two decades, complicated exercises in roots and radicals comprised two-thirds, or in two cases approached two-thirds of the exercises in this topic. The greatest emphasis was placed on them in 1820–29 and in 1890–99, in which decades approximately three out of every four exercises belonged to this group. There was a slight decrease in the first decade of the present century. This was followed by large decreases in the two succeeding decades. As a result less than half (47.52 per cent) of the exercises in roots and radicals are complicated in the present decade.

Different phases of roots and radicals are compared in Table 21. Radicals with fractional exponents have varied in amount from 11.99 to 23.66 per cent, the minimum being in the decade from 1830 to 1839 and the maximum in that from 1860 to 1869. The percentage in the present decade amounts to 15.31.

Roots of numbers increased from 8.60 per cent at the beginning of the period to 23.73 per cent in the decade of the fifties. In the next twenty years they decreased to 8.56 per cent. This was approximately the same percentage as in the first decade. Since the eighties, they have comprised between 9.45 and 14.04 per cent of the exercises in roots and radicals. In the present decade they amount to 13.79 per cent.

The percentage of exercises in the roots of algebraic expressions varied between 7.62 and 15.28 per cent in the nineteenth century,

the lowest percentage being in the decade of the seventies and the highest in that of the sixties. In the decades from 1870 to the present, the percentages have been less in every case than the minimum in the decades preceding this interval. This apparently indicates a tendency in the latter part of the period to lay less emphasis on this phase of the drill in roots and radicals. The percentage in the present decade amounts to 7.46, less than half the percentage in the sixties.

Exercises in radicals that are not included in the three groups just discussed are in almost all cases exercises in radicals with the radical sign. They have varied in amount from 46.74 per cent to 65.01 per cent. In the present decade they comprise 63.42 per cent of the exercises in roots and radicals.

Summary

Judged by the percentage of exercises devoted to them, roots and radicals received the most stress in the decades beginning in 1820, 1840, and 1890, and the least in those beginning in 1910 and 1920. The mean number of exercises in both 1910–19 and 1920–28 amounted to more than four times the number in the first decade of the period. The mean number was highest in 1900–1910, in which decade 426.3 exercises were, on the average, devoted to this topic.

The largest percentage of drill in roots and radicals has gradually changed from the middle to the last part of the course in elementary algebra, as judged by the textbooks examined. In addition, the exercises are less complicated now, the present decade being the first in which the simple exercises in this topic amount to more than half the total for the topic.

Except that the drill in roots of algebraic expressions has been less since the decade beginning in 1870, the various phases of roots and radicals have shown no consistent tendencies to change in relative emphasis. Roots of numbers received more drill than roots of algebraic expressions in nine of the eleven decades. In the present decade the former amount to 13.79 per cent and the latter to 7.46 per cent of the total exercises in roots and radicals.

IV

Fractions

The subject of this chapter is the development of fractions in mean number, percentage, distribution, complexity, and specific phases. The data on which the discussion is based and which show the development of fractions in these different points as revealed by the textbooks examined are given in the tables that follow.

Table 22 gives for each decade the mean number of exercises in fractions, and Table 23 the percentage of exercises in fractions. In Table 24 is shown the distribution of the exercises according to the fifth in which they appeared in their respective books.

The emphasis on complicated fractions in the different decades is given in Table 25. For this purpose the following exercises were classified as simple exercises:

1. Exercises containing less than four monomial or three binomial fractions, unless otherwise complicated.
2. Complex fractions containing one or two monomial or one binomial fraction in numerator, denominator, or both, unless included among the complicated exercises as defined below.

The complicated group includes:

1. All fractions with negative, fractional or literal exponents, or with radicals.
2. Fractions with more than two terms in numerator or denominator or with more than one compound term in numerator or denominator, etc.
3. Exercises in reducing fractions to infinite series, vanishing fractions, continued fractions, and complex fractions that are not included in the simple group.

In Table 26 are given the percentages of fractions that are exercises in complex fractions, exercises in reducing fractions to infinite series, and exercises in fractions not included in these two groups.

TABLE 22

MEAN NUMBER OF EXERCISES IN FRACTIONS PER BOOK, TOGETHER WITH A COMPARISON OF THE INCREASE OR DECREASE IN EACH DECADE

DECADE	MEAN NUMBER OF EXERCISES	CHANGE FROM PRECEDING DECADE		
		NUMBER OF EXERCISES	PER CENT INCREASE OR DECREASE	RANK IN PERCENTAGE OF INCREASE
1820	51.1*			
1830	72.4	+21.3	+41.7	1**
1840	91.6	+19.2	+26.5	3
1850	115.3	+23.7	+24.1	5
1860	143.3	+28.0	+24.3	4
1870	185.1	+41.8	+29.2	2
1880	213.5	+28.4	+15.3	7
1890	216.2	+ 2.7	+ 1.3	10*
1900	232.4	+16.2	+ 7.5	9
1910	257.4	+25.0	+10.8	8
1920	314.8**	+57.4	+22.3	6

*Minimum or lowest for the entire period.
**Maximum or highest for the entire period.

Read table as follows: In the decade beginning in 1830, the mean number of exercises in fractions amounted to 72.4 exercises, an increase of 21.3 exercises or a gain of 41.7 per cent over the decade of the twenties. The decade, 1830–39, ranked first in the percentage of increase in mean number of exercises in fractions.

TABLE 23

PERCENTAGE OF EXERCISES IN FRACTIONS, TOGETHER WITH A COMPARISON OF THE INCREASE OR DECREASE IN EACH DECADE

DECADE	EXERCISES IN FRACTIONS			
	PER CENT	CHANGE FROM PRECEDING DECADE	CHANGE FROM 1820	RANK IN PERCENTAGE OF EXERCISES
1820	13.29**			1**
1830	11.91	−1.38	−1.38	3
1840	10.70	−1.21	−2.59	5
1850	12.09	+1.39	−1.20	2
1860	9.49	−2.60	−3.80	7
1870	11.06	+1.57	−2.23	4
1880	9.77	−1.29	−3.52	6
1890	8.25	−1.52	−5.04	10
1900	8.48	+ .23	−4.81	9
1910	8.21*	− .27	−5.08	11*
1920	8.82	+ .61	−4.47	8

*Minimum or lowest for the entire period.
**Maximum or highest for the entire period.

Read table as follows: In the decade from 1840 to 1849, exercises in fractions amounted to 10.70 per cent of the total exercises for the decade. This was a decrease of 1.21 points from the percentage for 1830 to 1839, and of 2.59 points from that for 1820 to 1829. This decade ranked fifth in the emphasis placed on fractions. See Fig 10.

FIG. 10—PERCENTAGE OF EXERCISES IN FRACTIONS

TABLE 24

DISTRIBUTION OF THE EXERCISES IN FRACTIONS ACCORDING TO THE FIFTH IN WHICH THEY APPEARED IN THEIR RESPECTIVE BOOKS

DECADE	PER CENT OF FRACTIONS IN THE				
	FIRST FIFTH	SECOND FIFTH	THIRD FIFTH	FOURTH FIFTH	FIFTH FIFTH
1820	10.96	68.30**	18.20	2.35	.20*
1830	14.99	67.36**	3.75	.00*	13.91
1840	16.54	64.63**	15.94	2.89	.00*
1850	6.79	85.47**	1.12*	4.56	2.06
1860	.15	75.58**	24.26	.00*	.00*
1870	.64	69.83**	29.50	.00*	.03
1880	3.61	70.04**	26.35	.00*	.00*
1890	.14*	34.38	53.01**	11.58	.88
1900	.36*	48.40**	37.11	10.81	3.32
1910	.55*	12.33	61.94**	18.62	6.55
1920	2.71*	13.17	56.01**	24.32	3.79

*Minimum percentage for the decade.
**Maximum percentage for the decade.

Read table as follows: In the first decade, 10.96 per cent of the exercises in fractions appeared in the first fifth of their respective books; 68.30 per cent appeared in the second fifth; 18.20 per cent in the third fifth; 2.35 per cent in the fourth fifth; and .20 per cent in the last fifth.

The percentages given in this table have been computed according to the method described on page ix–x.

TABLE 25

PERCENTAGE RESPECTIVELY OF SIMPLE AND COMPLICATED EXERCISES IN FRACTIONS FOR EACH DECADE

DECADE	PER CENT OF EXERCISES	
	SIMPLE	COMPLICATED
1820	75.73	24.27
1830	79.27**	20.73*
1840	74.53	25.47
1850	74.27	25.73
1860	70.49	29.51
1870	75.61	24.39
1880	57.92	42.08
1890	46.94*	53.06**
1900	50.05	49.95
1910	60.87	39.13
1920	73.45	26.55

*Minimum percentage for the entire period.
**Maximum percentage for the entire period.

Read table as follows: In the first decade, 75.73 per cent of the exercises in fractions were simple exercises, and 24.27 per cent were complicated.

The types of exercises in fractions that compose the two groups—simple and complicated—are listed on page 43. See Fig 11.

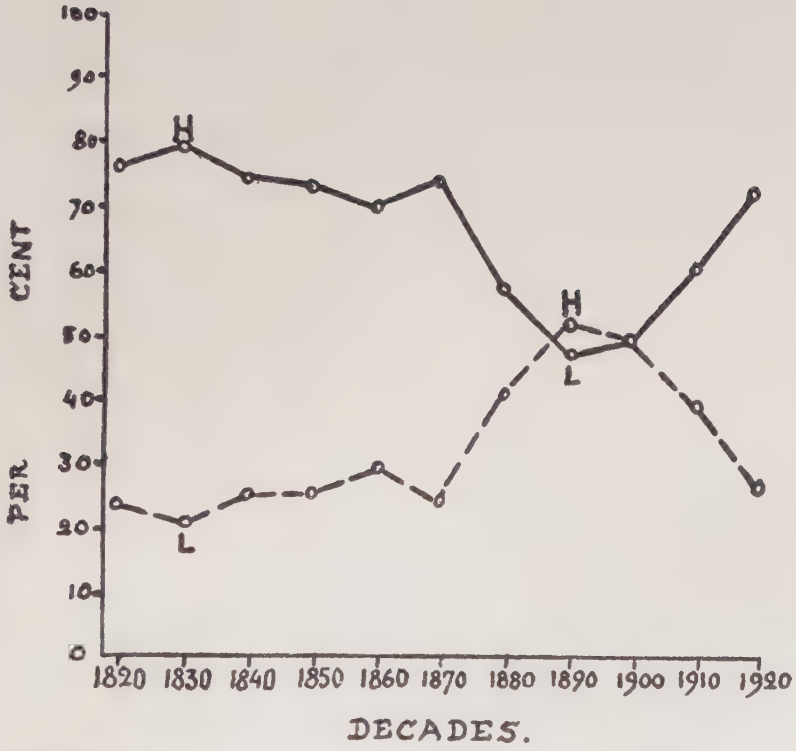


FIG. 11—COMPARISON OF SIMPLE AND COMPLICATED EXERCISES IN FRACTIONS

_____ Represents simple exercises in fractions.
_ _ _ Represents complicated exercises in fractions.

TABLE 26

PERCENTAGE OF EXERCISES IN FRACTIONS DEVOTED TO FRACTIONS TO BE REDUCED TO INFINITE SERIES, COMPLEX FRACTIONS, AND UNCLASSIFIED FRACTIONS, RESPECTIVELY

DECADE	PER CENT OF FRACTIONS DEVOTED TO		
	FRACTIONS TO BE REDUCED TO INFINITE SERIES	COMPLEX FRACTIONS	UNCLASSIFIED FRACTIONS
1820	2.15**	.00*	97.85
1830	.20	1.02	98.78**
1840	.82	1.47	97.71
1850	1.14	2.98	95.88
1860	.00*	3.40	96.60
1870	.00*	6.45	93.56
1880	.00*	6.63	93.37
1890	.00*	9.24**	90.76*
1900	.00*	7.93	92.07
1910	.00*	6.57	93.44
1920	.00*	5.09	94.91

*Minimum percentage for the entire period.
**Maximum percentage for the entire period.

Read table as follows: In the decade of the fifties, 1.14 per cent of the exercises in fractions were in the reduction of fractions to infinite series; 2.98 per cent were exercises in complex fractions; and 95.88 per cent were exercises in other kinds of fractions. See Fig. 12.

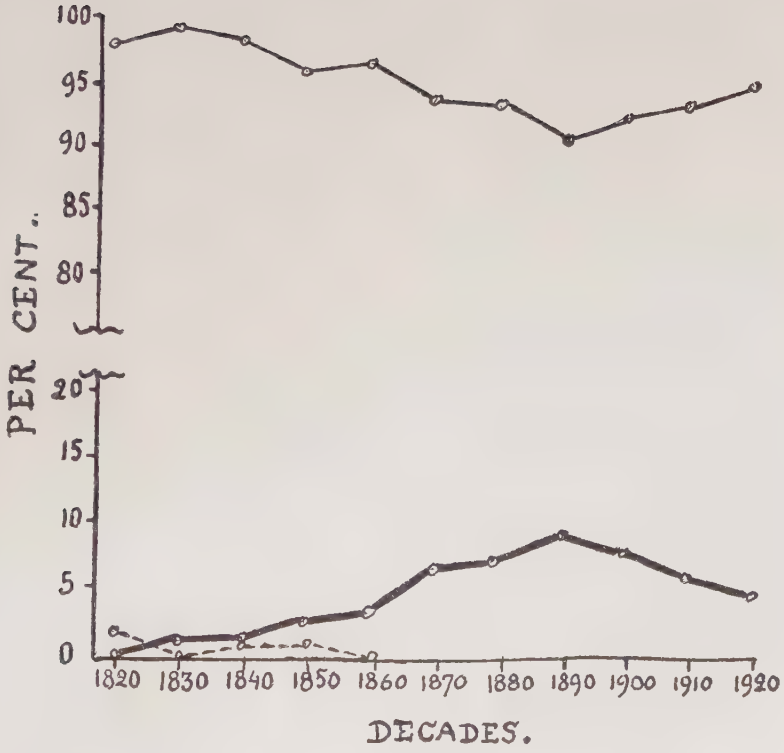


FIG. 12—COMPARISON OF COMPLEX FRACTIONS, FRACTIONS TO BE REDUCED TO INFINITE SERIES, AND UNCLASSIFIED FRACTIONS

——— Represents exercises in complex fractions.
– – – – Represents exercises in fractions to be reduced to infinite series.
——— Represents exercises in unclassified fractions.

Discussion

Table 22 shows that the mean number of exercises in fractions has increased from 51.1 in the first decade to 314.8 in the present decade. This is the only topic besides multiplication that has increased in every decade without at least one exception. The percentages of increase from decade to decade, however, have not been so large as in many other topics, the largest being 41.7 per cent in 1830–39.

Previous to 1850–59, the number of fractions per book amounted to less than 100, but in that decade, the number of fractions increased to 115.3 on the average. In the decades from 1880 to 1919 inclusive, the number exceeded 200. In the present decade it surpasses 300. The number in the present decade represents a gain of 22.3 per cent over the number for 1910–19.

The relative stress placed on fractions in comparison with the other topics of elementary algebra is shown in Table 23 and Figure 10. In the nineteenth century, the percentage decreased slowly from decade to decade with the exception of the decades beginning in 1850 and in 1870, each of which increased over the decade immediately preceding it. The total change in the last century amounted to 5.04 points, the percentage being 13.29 in the decade of the twenties and 8.25 in that of the nineties. The present century has shown little tendency to deviate from the latter percentage.

As is evident from Table 24, there has been a definite tendency to change the relative distribution of the drill in fractions. The change has been towards postponing much of the drill until later in the course, judging from the books examined. Preceding approximately 1890, the largest percentage of drill came in the second fifth, but since then it has generally been in the third fifth. The highest percentages in the first fifth occurred in the first three decades. Previous to the decade of the fifties, the percentage in this fifth varied between 10.96 and 14.99 per cent. In the present decade only 2.71 per cent of the exercises in fractions are in the first fifth. The percentage in the second fifth has also become smaller, the decrease beginning in the decade of the nineties. On the other hand, the third and fourth

fifths have increased, in the first case from 18.20 per cent in the first decade to 56.01 in the last decade, and in the second case from 2.35 per cent to 24.32 per cent in the same decades respectively.

Simple and complicated exercises in fractions are compared in Table 25 and Figure 11. Simple fractions increased from 75.73 per cent in the first decade to 79.27 per cent in the second decade. In the remaining decades of the century, the percentage decreased with the exception of the decade beginning in 1870. As Figure 11 shows, the decreases were slight up to and including the decade of the sixties. In the decade from 1880 to 1889 the simple exercises in fractions fell to 57.92 per cent, a difference of 17.69 points from the percentage in the seventies. In 1890–99 the percentage dropped an additional 10.98 points. In that decade only 46.94 per cent of the exercises belonged to the simpler group. The percentage increased slightly in the following decade, amounting to 50.05. In the decade beginning in 1910, it rose to 60.87. In the present decade it has increased further, the number of simple exercises totalling 73.45 per cent. Thus, there is apparently a tendency to return to the emphasis placed on simple exercises in the earlier decades of the nineteenth century.

Complicated exercises were most numerous in the decades between 1880 and 1909. During part of this time approximately one in every two fractions were of this kind. In the present decade, on the contrary, only slightly less than one fraction in four is complicated.

Table 26 and Figure 12 show that drill in complex fractions appeared in the decade from 1830 to 1839, in which years it amounted to 1.02 per cent of the total drill in fractions. Exercises in complex fractions increased throughout the rest of the century and in the decade of the nineties comprised 9.24 per cent. However, complex fractions have decreased since the beginning of the twentieth century and in the present decade amount to 5.09 per cent.

The reduction of fractions to infinite series received from .20 to 2.15 per cent of the total drill in fractions in the earlier part of the period. Since the decade of the sixties, however, it has

not been drilled upon. The largest amount of drill in it totalled 2.15 per cent and occurred in the decade from 1820 to 1829.

Summary

Exercises in fractions have increased in mean number but decreased in percentage of total exercises. The drill in the present decade comes somewhat later and is much simpler now than a few decades ago. Complex fractions in this decade comprise only a small part of the total number of exercises in fractions.

V

The General Equation and Formula. Substitution

The general equation or formula and substitution are the topics discussed in this chapter. The first two are considered together, as the formula is, in fact, a generalized equation. No sharp distinction can be drawn between the two. Books published in the earlier decades particularly, did not make any distinction between them. The tables and charts for general equations and formulas are given first, then those for substitution. After the latter, the results of both are discussed.

TABLE 27

Mean Number of Exercises in General Equations and Formulas Per Book, Together With a Comparison of the Increase or Decrease in Each Decade

Decade	Mean number of exercises	Change from preceding decade		
		Number of exercises	Per cent increase or decrease	Rank in percentage of increase
1820	40.9*			
1830	53.0	+ 12.1	+29.6	6
1840	78.8	+ 25.8	+48.7	2
1850	107.5	+ 28.7	+36.4	4
1860	131.8	+ 24.3	+22.6	7
1870	144.3	+ 12.5	+ 9.5	9
1880	201.2	+ 56.9	+39.4	3
1890	187.3	− 13.9	− 6.9	10*
1900	216.3	+ 29.0	+15.5	8
1910	289.4	+ 73.1	+33.8	5
1920	525.4**	+236.0	+81.5	1**

*Minimum or lowest for the entire period.
**Maximum or highest for the entire period.
Read table as follows: In the decade from 1830 to 1839, the mean number of exercises in general equations and formulas amounted to 53.0 exercises, an increase of 12.1 exercises or 29.6 per cent over 1820 to 1829. This decade ranked sixth in the percentage of increase in mean number of exercises in general equations and formulas.

Tables 27 and 28 give respectively the mean number and the percentage of exercises in general equations and formulas.

They also show the corresponding increases or decreases in each decade. Table 29 gives the percentage of exercises found in each fifth of the books. In Table 30, the application of general equations and formulas is compared with the exercises in their derivation.

Tables 31, 32, and 33 deal with substitution. Table 31 gives the mean number of exercises in this topic; Table 32, the percentage of exercises devoted to it; and Table 33, their distribution in the books.

TABLE 28

PERCENTAGE OF EXERCISES IN GENERAL EQUATIONS AND FORMULAS, TOGETHER WITH A COMPARISON OF THE INCREASE OR DECREASE IN EACH DECADE

DECADE	EXERCISES IN GENERAL EQUATIONS AND FORMULAS			
	PER CENT	CHANGE FROM PRECEDING DECADE	CHANGE FROM 1820	RANK IN PERCENTAGE OF EXERCISES
1820	10.64			2
1830	8.72	−1.92	−1.92	8
1840	9.20	+ .48	−1.44	6
1850	10.56	+1.36	− .08	3
1860	8.74	−1.82	−1.90	7
1870	8.62	− .12	−2.02	9
1880	9.21	+ .59	−1.43	5
1890	7.15*	−2.06	−3.49	11*
1900	7.89	+ .74	−2.75	10
1910	9.24	+1.35	−1.40	4
1920	14.73**	+5.49	+4.09	1**

*Minimum or lowest for the entire period.
**Maximum or highest for the entire period.

Read table as follows: In the decade from 1910 to 1919, exercises in general equations and formulas constituted 9.24 per cent of the total exercises for the decade. This was an increase of 1.35 points over the percentage for 1900 to 1909, but a decrease of 1.40 points from that for 1820 to 1829. This decade (1910–19) ranked fourth in the emphasis placed on general equations and formulas. See Fig. 13.

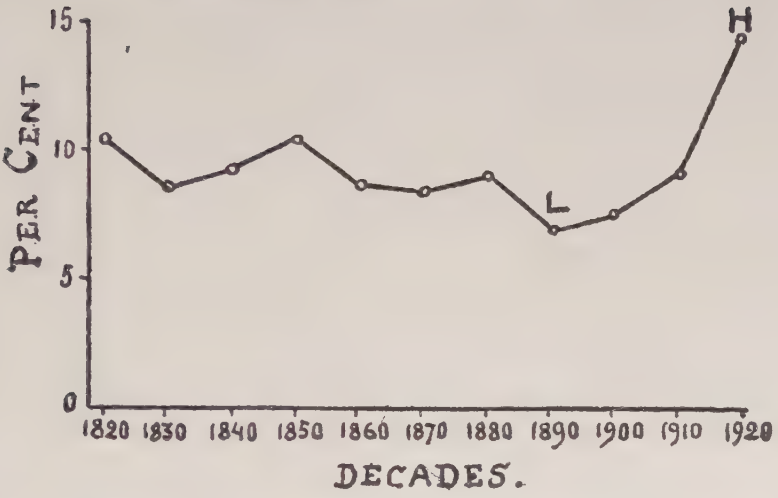


FIG. 13—PERCENTAGE OF EXERCISES IN GENERAL EQUATIONS AND FORMULAS

TABLE 29

DISTRIBUTION OF THE EXERCISES IN GENERAL EQUATIONS AND FORMULAS ACCORDING TO THE FIFTH IN WHICH THEY APPEARED IN THEIR RESPECTIVE BOOKS

DECADE	PER CENT OF GENERAL EQUATIONS AND FORMULAS IN THE				
	FIRST FIFTH	SECOND FIFTH	THIRD FIFTH	FOURTH FIFTH	FIFTH FIFTH
1820	.00*	27.38	9.29	35.94**	27.38
1830	.00*	7.28	4.99	37.20	50.54**
1840	.00*	11.94	23.94	18.48	45.65**
1850	1.24*	6.58	22.37	27.13	42.68**
1860	2.44*	6.07	17.44	14.32	59.73**
1870	1.47*	7.13	21.44	15.37	54.59**
1880	.57*	3.46	23.41	12.08	60.48**
1890	3.20*	4.01	17.15	17.18	58.46**
1900	4.41*	7.59	23.68	24.83	39.49**
1910	11.13*	17.32	16.35	27.31	27.88**
1920	21.40	13.57*	16.16	20.83	28.03**

*Minimum percentage for the decade.
**Maximum percentage for the decade.

Read table as follows: In the decade from 1850 to 1859, 1.24 per cent of the exercises in general equations and formulas appeared in the first fifth of their respective books; 6.58 per cent appeared in the second fifth; 22.37 per cent appeared in the third fifth; 27.13 per cent, in the fourth fifth; and 42.68 per cent, in the last fifth.

The percentages given in this table have been computed according to the method described on page ix–x.

TABLE 30

PERCENTAGE OF EXERCISES IN GENERAL EQUATIONS AND FORMULAS DEVOTED RESPECTIVELY TO THEIR SOLUTION OR DERIVATION, AND THEIR APPLICATION

DECADE	PER CENT OF GENERAL EQUATIONS AND FORMULAS DEVOTED TO	
	THEIR DERIVATION OR SOLUTION	THEIR APPLICATION
1820	61.12**	38.88*
1830	40.16	59.84
1840	44.68	55.32
1850	33.07*	66.93**
1860	45.00	55.00
1870	35.54	64.46
1880	43.56	56.44
1890	46.53	53.47
1900	53.00	47.00
1910	43.06	56.94
1920	35.85	64.15

*Minimum percentage for the entire period.
**Maximum percentage for the entire period.

Read table as follows: In the first decade, 61.12 per cent of the exercises in general equations or formulas dealt with their solution or derivation, and 38.88 per cent with their application. See Fig. 14.

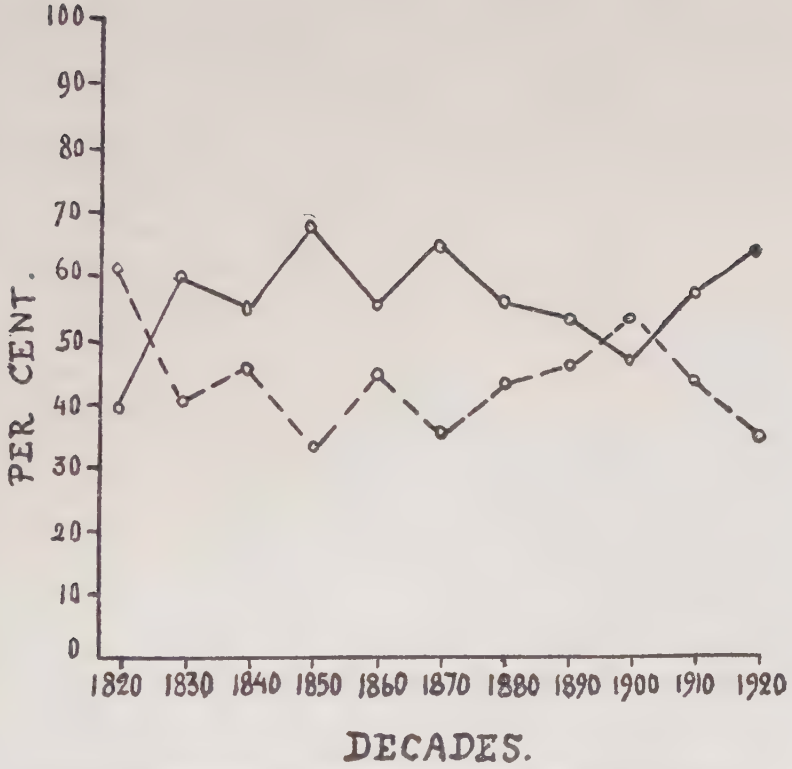


FIG. 14—COMPARISON OF THE APPLICATION AND THE DERIVATION OF GENERAL EQUATIONS AND FORMULAS

_____ Represents exercises in the application of general equations and formulas.

_ _ _ Represents exercises in the derivation or solution of general equations and formulas.

TABLE 31

MEAN NUMBER OF EXERCISES IN SUBSTITUTION PER BOOK, TOGETHER WITH A COMPARISON OF THE INCREASE OR DECREASE IN EACH DECADE

DECADE	MEAN NUMBER OF EXERCISES	CHANGE FROM PRECEDING DECADE		
		NUMBER OF EXERCISES	PER CENT INCREASE OR DECREASE	RANK IN PERCENTAGE OF INCREASE
1820	9.1			
1830	5.6*	− 3.5	− 38.5	10*
1840	16.5	+10.9	+194.6	1**
1850	10.7	− 5.8	− 35.2	9
1860	24.5	+13.8	+129.0	2
1870	26.2	+ 1.7	+ 6.9	6
1880	36.0	+ 9.8	+ 37.4	5
1890	64.8	+28.8	+ 80.0	3
1900	54.3	−10.5	− 16.2	8
1910	81.1	+26.8	+ 49.4	4
1920	81.8**	+ .7	+ .9	7

*Minimum or lowest for the entire period.
**Maximum or highest for the entire period.

Read table as follows: In the decade from 1830 to 1839, the mean number of exercises in substitution amounted to 5.6 exercises per book, a decrease of 3.5 exercises or 38.5 per cent from the first decade, 1820–29. The decade of the thirties ranked last in the percentage of increase in the mean number of exercises in substitution.

TABLE 32

PERCENTAGE OF EXERCISES IN SUBSTITUTION, TOGETHER WITH A COMPARISON OF THE INCREASE OR DECREASE IN EACH DECADE

DECADE	EXERCISES IN SUBSTITUTION			
	PER CENT	CHANGE FROM PRECEDING DECADE	CHANGE FROM 1820	RANK IN PERCENTAGE OF EXERCISES
1820	2.37			3
1830	.93*	−1.44	−1.44	11*
1840	1.93	+1.00	− .44	6
1850	1.12	− .81	−1.25	10
1860	1.63	+ .51	− .74	8
1870	1.56	− .07	− .81	9
1880	1.65	+ .09	− .72	7
1890	2.47	+ .82	+ .10	2
1900	1.98	− .49	− .39	5
1910	2.59**	+ .61	+ .22	1**
1920	2.29	− .30	− .08	4

*Minimum or lowest for the entire period.
**Maximum or highest for the entire period.

Read table as follows: In the decade from 1840 to 1849, exercises in substitution amounted to 1.93 per cent of the total exercises for the decade. This was an increase of one point over the percentage for the decade of the thirties, but a decrease of .44 points from that for the twenties. The decade of the forties ranked sixth in the emphasis placed on substitution.

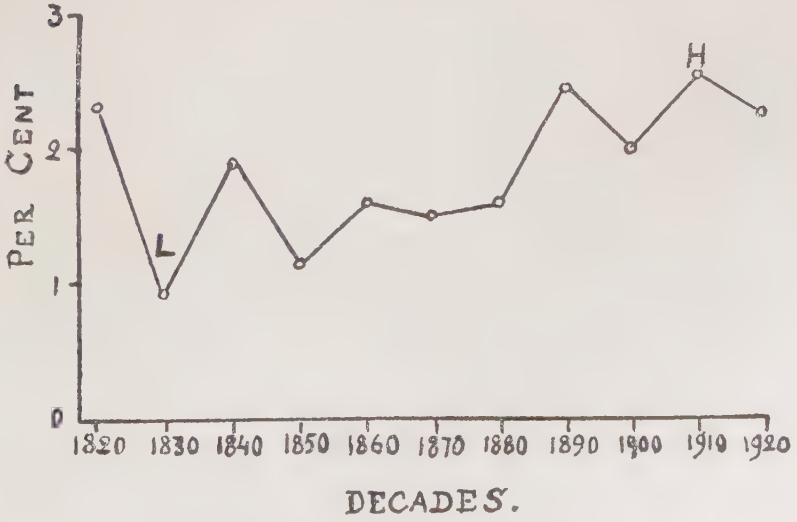


FIG. 15—PERCENTAGE OF EXERCISES IN SUBSTITUTION

TABLE 33

DISTRIBUTION OF EXERCISES IN SUBSTITUTION ACCORDING TO THE FIFTH IN WHICH THEY APPEARED IN THEIR RESPECTIVE BOOKS

DECADE	PER CENT OF EXERCISES IN SUBSTITUTION IN THE				
	FIRST FIFTH	SECOND FIFTH	THIRD FIFTH	FOURTH FIFTH	FIFTH FIFTH
1820	100.00**	0.00*	0.00*	0.00*	0.00*
1830	94.94**	2.53	0.00*	2.53	0.00*
1840	30.30	4.85	58.48**	6.36	0.00*
1850	96.11**	3.11	0.00*	.78	0.00*
1860	81.45**	4.98	12.67	0.00*	.91
1870	95.96**	1.35	.45	0.00*	2.25
1880	82.89**	9.32	3.48	1.95*	2.36
1890	85.93**	5.80	1.92	4.96	1.40*
1900	87.20**	3.56	4.18	4.18	.88*
1910	76.36**	11.82	3.90	6.53	1.39*
1920	64.77**	14.49	8.52	9.43	2.79*

*Minimum percentage for the decade.
**Maximum percentage for the decade.

Read table as follows: In the decade beginning in 1830, 94.94 per cent of the exercises in substitution appeared in the first fifth of their respective books; 2.53 per cent appeared in both the second and the fourth fifths. None appeared in the third and the last fifths.

The percentages given in this table have been computed according to the method described on page ix–x.

Discussion

Table 27 shows that the mean number of exercises in general equations and formulas has increased from decade to decade with the exception of the last decade of the nineteenth century. In the first decade, the mean number amounted to 40.9 exercises; in the fifties it totalled 107.5, and in the eighties, 201.2. In the decade of the nineties, it dropped to 187.3, but since the beginning of the century it has again exceeded 200 exercises. The greatest percentage of increase occurs in the present decade, the mean number of exercises being 81.5 per cent more than in 1910–19.

Although the mean number of exercises increased from the beginning of the period to the present with only one exception, the percentage of exercises devoted to general equations and formulas has shown varying tendencies. The percentage for the first decade—10.64—is the highest percentage in the topic

until the present decade is reached, according to Table 28 and Figure 13. In the decade beginning in 1830 the percentage decreased to 8.72. In the following decades it increased and in the decade of the fifties amounted to nearly the same percentage as existed in the first decade. After the beginning of the last half of the century, the percentage dropped from decade to decade with the exception of the eighties and reached the minimum in the decade from 1890 to 1899. From 7.15 per cent in the nineties, the percentage has increased to 14.73 in the present decade.

Table 29 shows the distribution of the exercises in general equations and formulas. Since the second decade of the period the largest percentage of these exercises has been in the last fifth. This is probably due in large measure to the application of algebraic formulas in topics appearing late in the course. However, the tendency has been to increase the percentage in the first fifth. In the first thirty years there were no exercises in general equations and formulas in this fifth. However, in the next few decades (1850–1889), the percentages varied between .57 and 2.44; but since approximately 1890, the percentage has steadily increased and in the present decade, amounts to 21.40. Although no exercises appeared in the beginning fifth in the first three decades, the percentages in the second fifth in both 1820–29 and 1840–49 are comparatively large. If the first two-fifths are combined, it may be seen that early drill in formulas was stressed nearly as much in 1820–29 (27.38 per cent) as it was in 1910–19 (28.45). Drill in the first two-fifths in the present decade amounts to 34.97 per cent.

That the tendency has recently been to distribute the drill more evenly in the different fifths may be seen by examining the last fifth in which, with the exception of the first decade, is found the maximum percentage. Of the eight decades of the past century, five have more than half their exercises in the last fifth, and all but the first have more than 40 per cent. On the other hand, the three decades of this century have smaller percentages in that fifth. The percentages for the decades beginning in 1900, 1910, and 1920, are respectively 39.49, 27.88, and 28.03.

According to the data in Table 30 and Figure 14 over half the

exercises in general equations and formulas have dealt with their application instead of their derivation or solution in every decade of the period with the exception of 1820–29 and 1900–09. In the decade of the twenties, the exercises in the application of general equations and formulas amounted to 38.88 per cent, but in the thirties they increased to 59.84 per cent. The percentage in the following decades varied between this latter figure and 66.93 and showed no tendency to increase or decrease until the decade beginning in 1880. In the interval between 1880 and 1909, however, the percentage decreased. In the decade from 1900 to 1909 it amounted to only 47.00 per cent. In the last two decades of the period (1910–1928) the percentage of exercises dealing with the application of general equations and formulas has increased, amounting in the present decade to 64.15 per cent.

As is seen in Figure 14, exercises in the solution or derivation of general equations and formulas were especially emphasized in the first decade of the period and in the first decade of the twentieth century. In these two decades 61.12 and 53.00 per cent, respectively, of the exercises dealt with the derivation of general equations and formulas. In all other decades, the percentages ranged from 33.07 to 46.53 per cent, and were consequently less than the percentages dealing with the application of formulas. At present the percentage amounts to 35.85.

The mean number of exercises in substitution was lowest (5.6) in the second decade of the period and highest (81.8) in the present decade. However, the relative emphasis given to substitution has shown no definite tendency to increase or decrease from decade to decade, the percentages varying between .93 and 2.59 per cent throughout the period. In the present decade exercises in substitution amount to 2.29 per cent. Tables 31 and 32 and Figure 15 show the change in mean number and the variation in percentage. Table 33 shows that the largest percentage of exercises in substitution has been in the first fifth in all the decades with the exception of the decade from 1840 to 1849, in which period of ten years the largest percentage came in the third fifth.

Summary

Formulas and general equations are stressed in the present decade more than at any time previously. They are also distributed more evenly through the various fifths, judging from the textbooks examined. Nearly two-thirds of the exercises are in the application of formulas and general equations and slightly over one-third are in their derivation. This is not a new departure, however, as in only two previous decades has the percentage in the application of formulas and general equations been lower than that in their derivation or solution. The mean number of exercises in general equations and formulas now amounts to 525.4 and the percentage to 14.73.

VI

Multiplication

The purpose of this chapter is to show the development of multiplication in the elementary algebra course, as revealed by the exercises in multiplication included in the textbooks examined.

Table 34 shows the mean number of exercises for each decade. Table 35 shows the percentage of exercises devoted to the topic. In both Table 34 and Table 35, the change from one decade to another is indicated. The distribution of the exercises according to the fifth in which they appeared in their respective books is given in Table 36.

In Table 37 are compared the percentages of simple and complicated exercises. For this purpose, the exercises in multiplication were divided into two groups, (1) simple and (2) complicated, as follows:

1. Exercises with monomial or binomial multipliers unless otherwise included in the second group. Exercises in special products of the following types: $(A+B)(A-B)$, $K(A\pm B)^2$, and $(AX\pm BY)(CX\pm DY)$, unless otherwise included in the second group.

2. All exercises with fractional, literal or negative exponents, radicals or compound terms.

Exercises with multipliers of three or more terms.

Exercises in multiplication by detached coefficients.

Exercises in the binomial theorem.

Exercises in special products requiring the grouping of terms, and those not in the first group.

The relative development of short multiplication, long multiplication, the binomial theorem, and special products is shown in Table 38. The percentage for each group is given in the order in which it has been named above. The exercises with monomial multipliers form the first group; the exercises with multipliers of two, three, or more terms, with the exception of exercises in special products, form the second group; exercises in powers of expressions, developed separately from the other exercises in

multiplication and not included in either of the remaining two classifications, form the third group; exercises in the binomial theorem and in special products form respectively the fourth and the fifth groups.

TABLE 34

MEAN NUMBER OF EXERCISES IN MULTIPLICATION PER BOOK, TOGETHER WITH A COMPARISON OF THE INCREASE OR DECREASE IN EACH DECADE

DECADE	MEAN NUMBER OF EXERCISES	CHANGE FROM PRECEDING DECADE		
		NUMBER OF EXERCISES	PER CENT INCREASE OR DECREASE	RANK IN PERCENTAGE OF INCREASE
1820	37.1*			
1830	78.1	+41.0	+110.5	1**
1840	104.0	+25.9	+ 33.2	3
1850	110.0	+ 6.0	+ 5.8	8
1860	167.0	+57.0	+ 51.8	2
1870	192.4	+25.4	+ 1.0	9
1880	239.3	+46.9	+.23.4	5
1890	311.6	+72.3	+ 30.2	4
1900	314.1	+ 2.5	+ .8	10*
1910	386.8	+72.7	+ 23.2	6
1920	435.5**	+48.7	+ 12.6	7

*Minimum or lowest for the entire period.
**Maximum or highest for the entire period.

Read table as follows: In the decade from 1830 to 1839, the mean number of exercises in multiplication amounted to 78.1 exercises per book, an increase of 41.0 exercises or 110.5 per cent over the mean number for the decade of the twenties. The decade of the thirties ranked first in the percentage of increase in mean number of exercises in multiplication.

TABLE 35

PERCENTAGE OF EXERCISES IN MULTIPLICATION, TOGETHER WITH A COMPARISON OF THE INCREASE OR DECREASE IN EACH DECADE

DECADE	EXERCISES IN MULTIPLICATION			
	PER CENT	CHANGE FROM PRECEDING DECADE	CHANGE FROM 1820	RANK IN PERCENTAGE OF EXERCISES
1820	9.65*			11*
1830	12.85**	+3.20	+3.20	1**
1840	12.15	− .70	+2.50	4
1850	11.53	− .62	+1.88	6
1860	11.06	− .47	+1.41	9
1870	11.50	+ .44	+1.85	7
1880	10.96	− .54	+1.31	10
1890	11.89	+ .93	+2.24	5
1900	11.46	− .43	+1.81	8
1910	12.35	+ .89	+2.70	2
1920	12.21	− .14	+2.56	3

*Minimum or lowest for the entire period.
**Maximum or highest for the entire period.

Read table as follows: In the decade from 1840 to 1849, the exercises in multiplication amounted to 12.15 per cent of the total exercises for the decade. This was a decrease of .70 points from the percentage for the decade of the thirties, but an increase of 2.50 points over that for the twenties. The decade of the forties ranked fourth in the emphasis on multiplication.

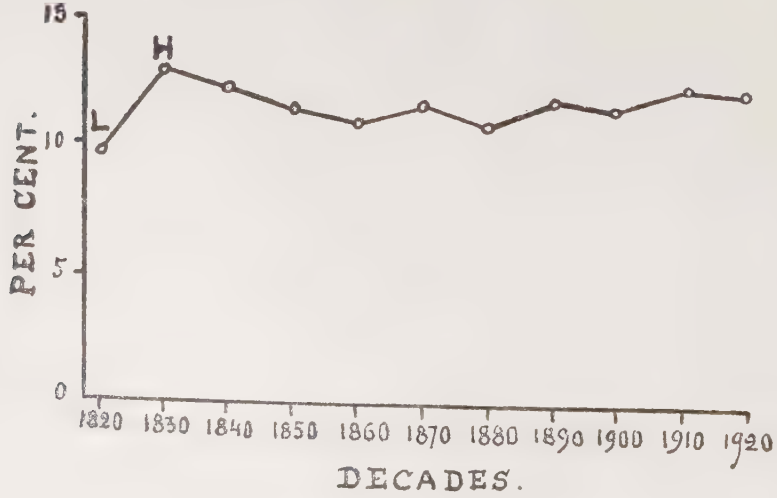


FIG. 16—PERCENTAGE OF EXERCISES IN MULTIPLICATION

TABLE 36

DISTRIBUTION OF EXERCISES IN MULTIPLICATION ACCORDING TO THE FIFTH IN WHICH THEY APPEARED IN THEIR RESPECTIVE BOOKS

DECADE	PER CENT OF EXERCISES IN MULTIPLICATION IN THE				
	FIRST FIFTH	SECOND FIFTH	THIRD FIFTH	FOURTH FIFTH	FIFTH FIFTH
1820	61.73**	21.56	13.21	.00*	3.50
1830	58.77**	13.89	21.21	5.12	1.01*
1840	52.88**	20.68	16.01	9.52	.91*
1850	46.89**	21.93	16.59	12.61	1.97*
1860	43.91**	21.56	13.64	20.89	.00*
1870	56.25**	11.07	19.50	9.32	3.85*
1880	56.21**	15.78	18.49	9.44	.08*
1890	52.91**	22.16	11.46	12.25	1.23*
1900	42.63**	30.79	13.81	11.39	1.38*
1910	20.69	48.92**	16.76	9.56	4.07*
1920	18.27	45.14**	21.93	9.92	4.75*

*Minimum percentage for the decade.
**Maximum percentage for the decade.

Read table as follows: In the decade beginning in 1830, 58.77 per cent of the exercises in multiplication appeared in the first fifth of their respective books; 13.89 per cent appeared in the second fifth; 21.21, 5.12, and 1.01 per cent appeared, respectively, in the third, fourth, and last fifths.

The percentages given in this table have been computed according to the method described on page ix–x.

TABLE 37

PERCENTAGE OF SIMPLE AND COMPLICATED EXERCISES IN MULTIPLICATION FOR EACH DECADE

DECADE	PER CENT OF EXERCISES	
	SIMPLE	COMPLICATED
1820	73.59	26.42
1830	76.65	23.35
1840	77.65	22.35
1850	72.84	27.16
1860	69.64	30.36
1870	76.09	23.91
1880	67.67*	32.33**
1890	68.85	31.15
1900	67.77	32.23
1910	81.69	18.31
1920	91.08**	8.92*

*Minimum percentage for the entire period.
**Maximum percentage for the entire period.

Read table as follows: In the first decade, 73.59 per cent of the exercises in multiplication were simple exercises and 26.42 per cent were complicated.

The types of exercises in multiplication that compose the two groups—simple and complicated—are listed on page 64. See Fig. 17.

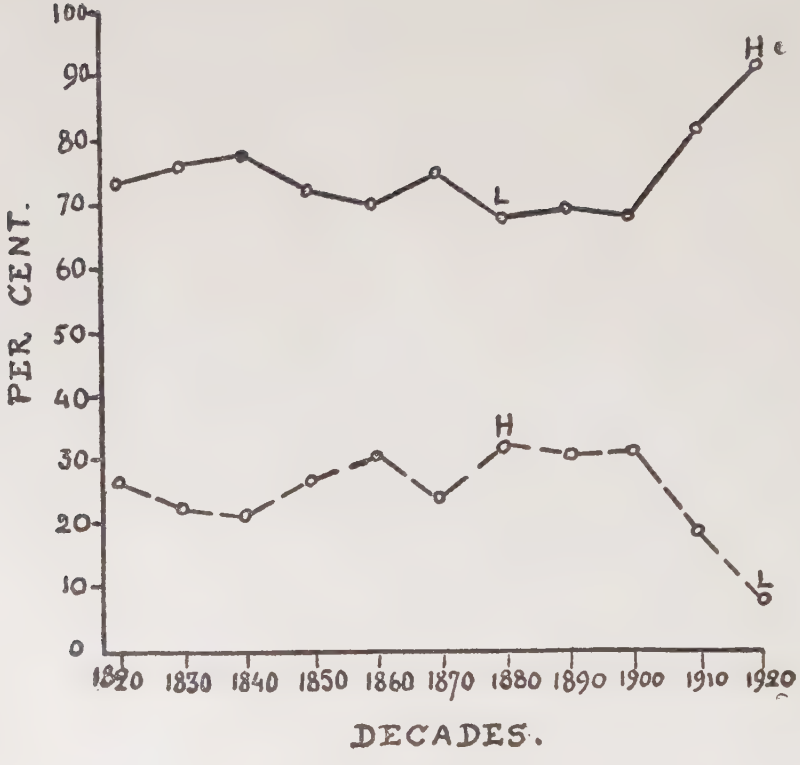


FIG. 17—COMPARISON OF SIMPLE AND COMPLICATED EXERCISES IN MULTIPLICATION

_____ Represents simple exercises in multiplication.
_ _ _ Represents complicated exercises in multiplication.

TABLE 38

PERCENTAGE OF EXERCISES IN MULTIPLICATION DEVOTED TO SHORT MULTIPLICATION, LONG MULTIPLICATION, INVOLUTION, BINOMIAL THEOREM, AND SPECIAL PRODUCTS, RESPECTIVELY

DECADE	PER CENT OF EXERCISES IN MULTIPLICATION DEVOTED TO				
	SHORT MULTIPLICATION	LONG MULTIPLICATION	INVOLUTION	BINOMIAL THEOREM	SPECIAL PRODUCTS
1820	42.86	26.15**	20.75	7.28	2.96*
1830	42.81	25.62	17.51	9.95	4.11
1840	45.94**	22.46	18.09	8.62	4.88
1850	32.88	22.03	23.48**	8.22	13.40
1860	28.45	17.18	15.18	16.45**	22.73
1870	32.18	18.29	19.55	9.20	20.78
1880	29.82	16.64	13.92	10.55	29.08
1890	27.54	17.28	13.73	7.99	33.46
1900	25.73*	15.83	12.04	7.86	38.54
1910	32.34	14.28*	8.42	3.57	41.38**
1920	40.85	14.80	5.71*	.27*	38.37

*Minimum percentage for the entire period.
**Maximum percentage for the entire period.

Read table as follows: In the first decade, 42.86 per cent of the exercises in multiplication were in short multiplication; 26.15 per cent, in long multiplication; 20.75 per cent, in involution; 7.28 per cent, in the binomial theorem; and 2.96 per cent, in special products. See Fig. 18.

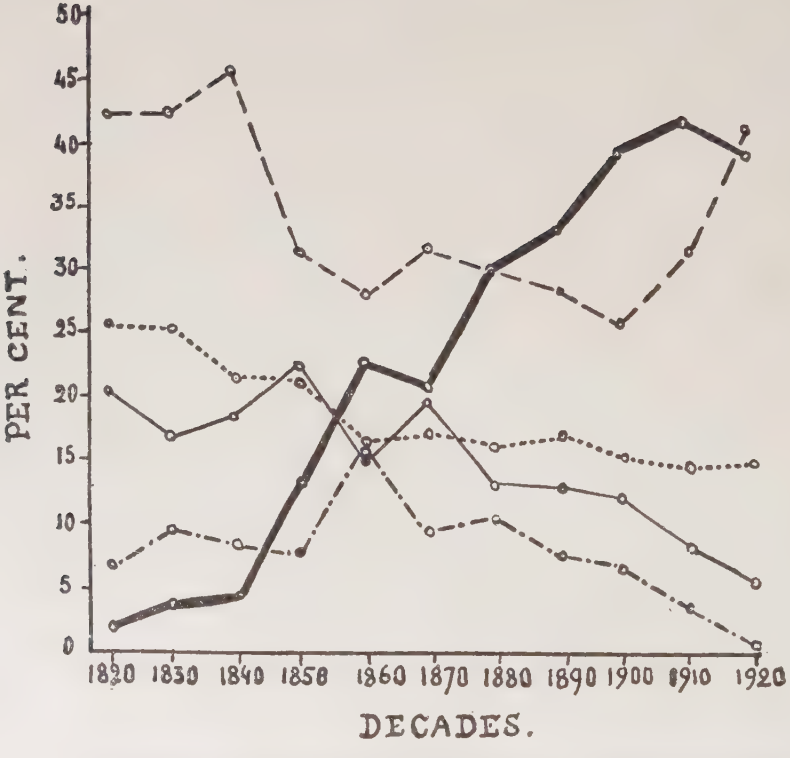


FIG. 18—COMPARISON OF SHORT MULTIPLICATION, LONG MULTIPLICATION, INVOLUTION, THE BINOMIAL THEOREM, AND SPECIAL PRODUCTS

_ _ _ _ Represents exercises in short multiplication.
. . . . Represents exercises in long multiplication.
_____ Represents exercises in involution.
_ . _ Represents exercises in the binomial theorem.
_____ Represents exercises in special products.

Discussion

Table 34 shows that the mean number of exercises in multiplication has increased from 37.1 in the first decade to 435.5 in the present decade. Multiplication is one of the two topics that has not decreased in at least one decade. The largest percentage of increase occurred in the second decade of the period when the mean number more than doubled. However, it was not until the decade of the forties that the mean number exceeded 100 exercises. In the eighties it surpassed 200, and in the first decade of the present century, 300.

The percentage of exercises in multiplication for each decade is shown in Table 35 and Figure 16. The minimum occurred in 1820–29 and amounted to 9.65 per cent. The maximum—12.85 per cent—came in the next decade. Beginning in the forties, the percentage decreased slightly for a few decades, and then varied until the end of the period. It has, however, never radically changed from the percentage for 1830 to 1839. At present, 12.21 per cent of the exercises in elementary algebra in the books examined are in multiplication.

As Table 36 shows, the largest percentage of exercises in this topic appeared in the first fifth until the last two decades. Since the 1910 decade it has been in the second fifth. The third fifth has had between 11.46 and 21.93 per cent of the drill. The last two fifths generally had less. There has been no clear tendency to increase or decrease the drill in the last three fifths, but the tendency to decrease the drill in the first fifth is clearly evident from the beginning of the period. In this fifth, the exercises decreased from 61.73 per cent in the twenties to 43.91 per cent in the sixties. In the seventies, the percentage increased to 56.25, but in the next decade the tendency to decrease began again. Since the decade of the eighties, this tendency to decrease has continued with the result that in the present decade the exercises in the first fifth amount to only 18.27 per cent.

Table 37 and Figure 17 show a striking change in the relative simplicity and complexity of multiplication exercises in the last two decades. Previous to the second decade of the present century, the number of simple exercises varied between 67.67

and 77.65 per cent, the highest percentages being in the first four decades and in the decade of the seventies. In 1910–19, however, simple exercises in this topic amounted to 81.69 per cent of the total number of exercises in multiplication. In the present decade they total 91.08 per cent. Complicated exercises, consequently, were most numerous in the decades between 1880 and 1909. During this interval nearly one exercise in three was complicated.

In the nineteenth century, short multiplication was stressed more in the decades preceding 1850 than in those following it. Table 38 shows that in the first three decades short multiplication exceeded 40 per cent; that in the decades between 1850 and 1879 inclusive, it varied between 32.88 and 28.45 per cent; and that beginning with the decade of the eighties it decreased for several decades, the minimum being reached in 1900–09. From 25.73 per cent in the 1900 decade, it has increased to 40.85 per cent in the present decade.

As Figure 18 shows, the trend in long multiplication has been, on the whole, to decrease the percentage of exercises in it. In the first decade of the period, the percentage amounted to 26.15. In the present decade, it is 14.80. The maximum percentage occurred in the decade from 1820 to 1829, and the minimum in that beginning in 1910. The percentage for the decade from 1910 to 1919, however, is only .52 points less than the percentage for the present decade.

Exercises in both involution and the binomial theorem have decreased steadily in percentage in the last few decades, involution since the eighties and the binomial theorem since the nineties. At present, involution comprises 5.71 per cent of the exercises in multiplication, and the binomial theorem .27 per cent. Preceding 1880, the percentage in the former varied between 15.18 and 23.48 per cent. Preceding 1890, the percentage in the latter ranged from 7.28 to 16.45 per cent.

Exercises in special products amounted to 2.96 per cent in the first decade (1820–29). In 1910–19 they totalled 41.38 per cent, the maximum. As Figure 18 shows, the percentage increased continually throughout the intervening interval, with the exception of the seventies,[1] which showed a slight decrease from

the decade immediately preceding. In the present decade, exercises in special products make up 38.37 per cent of the exercises in multiplication. This is a decrease of 3.01 points from the percentage for 1910–19.

Summary

Multiplication has increased in mean number, but the percentage of total exercises devoted to it has varied only slightly since the decade of the thirties. Drill in this topic now comes to a large extent later in the course, the stress being on the simpler exercises of short multiplication and special products. The emphasis on long multiplication, involution, the binomial theorem, and complicated exercises of all types has declined.

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[1] This decade was reactionary in tendency in the majority of the topics of the course.

VII

Addition and Subtraction

The subject of this chapter is the development of positive and negative numbers, and their addition and subtraction. The discussion is preceded by Tables 39 to 43, which deal with various aspects of the development of this topic.

TABLE 39

Mean Number of Exercises in Addition and Subtraction Per Book Together With a Comparison of the Increase or Decrease in Each Decade

Decade	Mean number of exercises	Change from preceding decade		
		Number of exercises	Per cent increase or decrease	Rank in percentage of increase
1820	34.6*			
1830	62.5	+27.9	+80.6	1**
1840	56.5	− 6.0	− 9.6	10*
1850	68.1	+11.6	+20.5	4
1860	90.0	+21.9	+32.1	3
1870	107.1	+17.1	+19.0	5
1880	121.1	+14.0	+13.1	8
1890	128.6	+ 7.5	+ 6.2	9
1900	153.0	+24.4	+19.0	6
1910	252.8	+99.8	+65.2	2
1920	293.1**	+40.3	+15.9	7

*Minimum or lowest for the entire period.
**Maximum or highest for the entire period.
Read table as follows: In the decade from 1830 to 1839, the mean number of exercises in addition and subtraction amounted to 62.5 exercises per book, an increase of 27.9 exercises or 80.6 per cent over the mean number for the decade from 1820 to 1829. The decade of the thirties ranked first in the percentage of increase in mean number of exercises in addition and subtraction.

The mean number and percentage of exercises in positive and negative numbers, and their addition and subtraction, are given in Tables 39 and 40 respectively. These tables indi-

cate also the change from decade to decade. Table 41 deals with the distribution of the exercises in the books in which they were found.

Table 42 compares the percentage of simple and complicated exercises. All exercises with negative, fractional, or literal exponents, radicals, or compound terms were considered complicated.

In Table 43 the exercises in addition are compared with those in subtraction.

TABLE 40

PERCENTAGE OF EXERCISES IN ADDITION AND SUBTRACTION, TOGETHER WITH A COMPARISON OF THE INCREASE OR DECREASE IN EACH DECADE

DECADE	EXERCISES IN ADDITION AND SUBTRACTION			
	PER CENT	CHANGE FROM PRECEDING DECADE	CHANGE FROM 1820	RANK IN PERCENTAGE OF EXERCISES
1820	9.00			2
1830	10.28**	+1.28	+1.28	1**
1840	6.60	−3.68	−2.40	6
1850	7.14	+ .54	−1.86	5
1860	5.96	−1.18	−3.04	8
1870	6.40	+ .44	−2.60	7
1880	5.54	− .86	−3.46	10
1890	4.91*	− .63	−4.09	11*
1900	5.58	+ .67	−3.42	9
1910	8.07	+2.49	− .93	4
1920	8.22	+ .15	− .78	3

*Minimum or lowest for the entire period.
**Maximum or highest for the entire period.

Read table as follows: In the decade from 1910 to 1919, exercises in addition and subtraction amounted to 8.07 per cent of the total exercises for the decade. This was an increase of 2.49 points over the percentage for the preceding decade, 1900–09, but a decrease of .93 points from the percentage for the first decade, 1820–29. This decade, 1910–19, ranked fourth in the emphasis on addition and subtraction. See Fig. 19.

FIG. 19—PERCENTAGE OF EXERCISES IN ADDITION AND SUBTRACTION

TABLE 41

DISTRIBUTION OF EXERCISES IN ADDITION AND SUBTRACTION ACCORDING TO THE FIFTH IN WHICH THEY APPEARED IN THEIR RESPECTIVE BOOKS

DECADE	PER CENT OF EXERCISES IN ADDITION AND SUBTRACTION IN THE				
	FIRST FIFTH	SECOND FIFTH	THIRD FIFTH	FOURTH FIFTH	FIFTH FIFTH
1820	93.93**	3.18	2.89	.00*	.00*
1830	88.80**	5.14	5.03	1.03	.00*
1840	91.06**	5.04	3.72	.18	.00*
1850	90.70**	7.47	1.78	.06	.00*
1860	88.15**	11.85	.00*	.00*	.00*
1870	98.57**	.99	.44	.00*	.00*
1880	87.49**	12.43	.08	.00*	.00*
1890	92.74**	7.19	.07	.00*	.00*
1900	92.84**	6.90	.03*	.15	.09
1910	87.53**	11.03	.03*	1.34	.08
1920	54.40**	39.46	1.02	5.12	.00*

*Minimum percentage for the decade.
**Maximum percentage for the decade.

Read table as follows: In the decade beginning in 1820, 93.93 per cent of the exercises in addition and subtraction appeared in the first fifth of their respective books; 3.18 per cent appeared in the second fifth; and 2.89 per cent appeared in the third fifth. There were none in the last two fifths.

The percentages given in this table have been computed according to the method described on page ix–x.

TABLE 42

PERCENTAGE OF SIMPLE AND COMPLICATED EXERCISES IN ADDITION AND SUBTRACTION FOR EACH DECADE

DECADE	PER CENT OF EXERCISES	
	SIMPLE	COMPLICATED
1820	81.79*	18.21**
1830	90.31	9.69
1840	88.75	11.25
1850	92.00	8.00
1860	93.75	6.25
1870	89.69	10.31
1880	95.11	4.89
1890	94.98	5.02
1900	93.94	6.06
1910	96.15	3.85
1920	97.37**	2.63*

*Minimum percentage for the entire period.
**Maximum percentage for the entire period.

Read table as follows: In the first decade, 81.79 per cent of the exercises in addition and subtraction were simple exercises and 18.21 per cent were complicated See Fig 20.

The types of exercises in addition and subtraction that compose the two groups—simple and complicated—are listed on page 75.

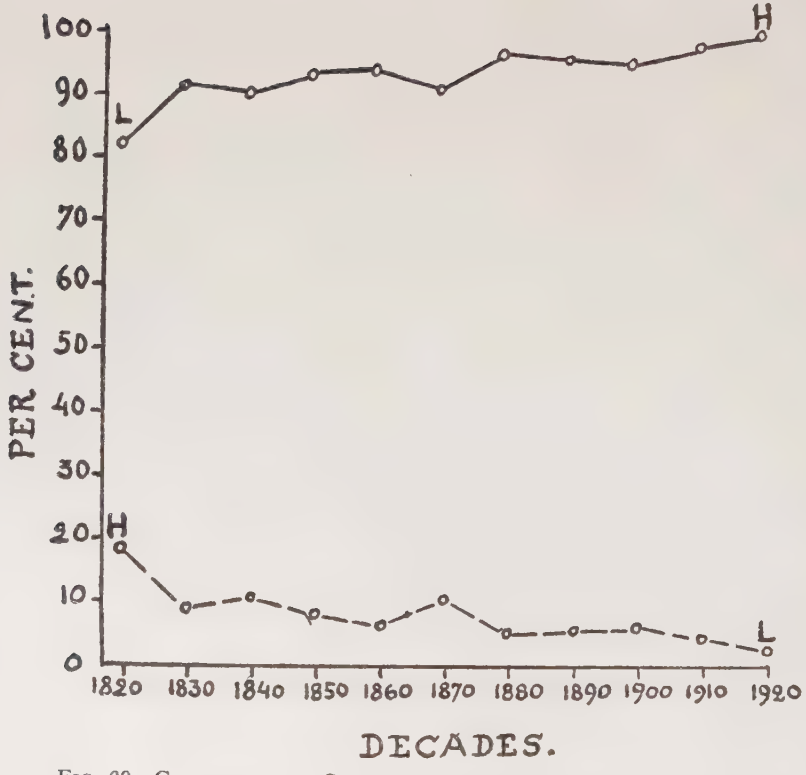

FIG. 20—COMPARISON OF SIMPLE AND COMPLICATED EXERCISES IN ADDITION AND SUBTRACTION

——— Represents simple exercises in addition and subtraction.
— — — Represents complicated exercises in addition and subtraction.

TABLE 43

PERCENTAGE OF EXERCISES IN ADDITION AND SUBTRACTION DEVOTED TO ADDITION AND SUBTRACTION, RESPECTIVELY

DECADE	PER CENT OF EXERCISES IN ADDITION AND SUBTRACTION DEVOTED TO	
	ADDITION	SUBTRACTION
1820	60.40	39.60
1830	57.18	42.82
1840	63.50**	36.50*
1850	59.08	40.92
1860	51.06*	48.94**
1870	59.46	40.54
1880	56.87	43.13
1890	61.09	38.91
1900	55.61	44.39
1910	55.80	44.20
1920	53.08	46.92

*Minimum percentage for the entire period.
**Maximum percentage for the entire period.

Read table as follows: In the first decade, 60.40 per cent of the exercises in addition and subtraction were in addition and 39.60 per cent were in subtraction. See Fig. 21.

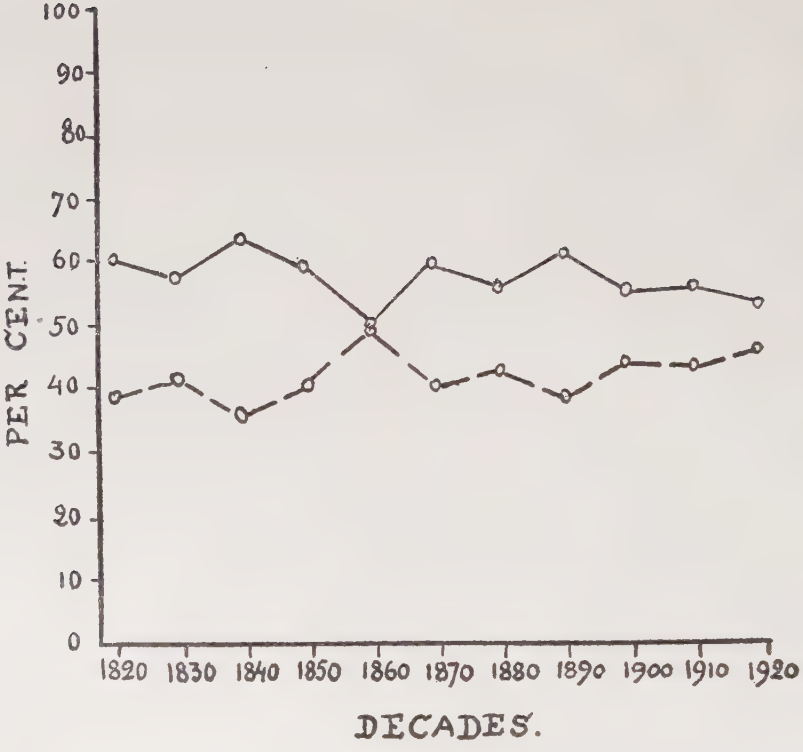

FIG. 21—COMPARISON OF ADDITION AND SUBTRACTION

——— Represents exercises in addition.
— — — Represents exercises in subtraction.

Discussion

Table 39 shows that the mean number of exercises in addition and subtraction has increased from the beginning of the period to the present, with the exception of the decade from 1840 to 1849. The minimum, occurring in the first decade, amounted to 34.6 exercises; the maximum, in the present decade, totals 293.1 exercises. The largest percentage of increase, 80.6 per cent, occurred in the decade from 1830 to 1839; the second largest, 65.2 per cent, in the decade beginning in 1910.

Table 40 shows that the percentage of exercises in addition and subtraction was highest in 1830–39 in which decade 10.28 per cent of all the exercises in the course were devoted to these two topics combined. In the remainder of the century, the percentage varied from decade to decade, although always on a lower level. In the decade of the nineties, 4.91 per cent of the exercises dealt with addition and subtraction. This was the minimum for the period. Since the beginning of the twentieth century, the percentage has increased and in the present decade amounts to 8.22. Figure 19 shows these changes graphically.

The distribution of the exercises in Table 41 shows that the largest percentage of exercises in addition and subtraction has been in the first fifth throughout the period. The percentage in this fifth, however, is much smaller in the present decade than it was previously. On the other hand, the percentage in the second fifth has increased. There has been occasionally a little drill in the last two-fifths. This is mostly in exercises with literal exponents and it forms a very small percentage of the total.

The data in Table 42 and the graph in Figure 20 show that complicated exercises in addition and subtraction were stressed most preceding 1880. The largest percentage—18.21—came in the first decade. The minimum number of complicated exercises is found in the present decade and amounts to 2.63 per cent. Simple exercises have ranged from 81.79 to 97.37 per cent, the minimum being in the first decade and the maximum in the last.

Table 43 and the corresponding graph show that there usually has been from 10 to 20 per cent more drill in addition than in subtraction. In the present decade the difference between them is approximately 6 per cent, addition receiving 53.08 per cent

of the drill and subtraction 46.92 per cent. Subtraction received more drill in the decade from 1860 to 1869 than in any other decade. Addition received the highest percentage of drill in the last decade of the nineteenth century.

Summary

Addition and subtraction received the least emphasis in the second half of the nineteenth century, and the most in the decades preceding that time. In the present decade they are stressed more than they were in the last decades of the past century. As in most other topics, the mean number of exercises has increased from decade to decade with only one exception in the entire period. The percentage of complicated exercises, never very large, is smallest in the present decade. The exercises have been generally quite evenly distributed between addition and subtraction.

VIII

Division

In the first part of this chapter are given the tables and graphs showing the development of division in the elementary algebra course as revealed by the textbooks examined. These are discussed in the latter part of the chapter.

Table 44 gives the mean number of exercises in division, together with a comparison of the increase or decrease in each decade. Table 45 presents the percentage of exercises in division for each decade, and the corresponding changes throughout the period. The distribution of the exercises according to the fifth in which they appeared in their respective books is found in Table 46.

TABLE 44

Mean Number of Exercises in Division Per Book, Together With a Comparison of the Increase or Decrease in Each Decade

Decade	Mean number of exercises	Change from preceding decade		
		Number of exercises	Per cent increase or decrease	Rank in percentage of increase
1820	27.0*			
1830	53.2	+26.2	+97.0	1**
1840	59.0	+ 5.8	+10.9	7
1850	61.7	+ 2.7	+ 4.6	8
1860	70.3	+ 8.6	+13.9	6
1870	94.2	+23.9	+34.0	2
1880	120.5	+26.3	+27.9	3
1890	137.3	+16.8	+13.9	5
1900	126.0	−11.3	− 8.3	10*
1910	158.6	+32.6	+25.9	4
1920	165.6**	+ 7.0	+ 4.4	9

*Minimum or lowest for the entire period.
**Maximum or highest for the entire period.

Read table as follows: In the decade from 1830 to 1839, the mean number of exercises in division amounted to 53.2 exercises per book, an increase of 26.2 exercises or 97.0 per cent over the mean number for the preceding decade. The decade of the thirties ranked first in the percentage of increase in the mean number of exercises in division.

Simple and complicated exercises in division are compared in Table 47. For this purpose the exercises with monomial divisors or with binomial divisors and quotients unless otherwise complicated were considered simple exercises. The complicated group includes: the exercises with literal, fractional, or negative exponents; the exercises with radicals or compound terms; the exercises with more than two terms in the divisor or with more than two terms in the quotient unless the divisor is a monomial; the exercises in detached coefficients, synthetic division, etc.

Long division, short division, and special quotients are compared in Table 48. Short division comprises the exercises with monomial divisors; long division, those with divisors of two or more terms, except those in "Special Quotients".

TABLE 45

PERCENTAGE OF EXERCISES IN DIVISION, TOGETHER WITH A COMPARISON OF THE INCREASE OR DECREASE IN EACH DECADE

DECADE	EXERCISES IN DIVISION			
	PER CENT	CHANGE FROM PRECEDING DECADE	CHANGE FROM 1820	RANK IN PERCENTAGE OF EXERCISES
1820	7.02			2
1830	8.75**	+1.73	+1.73	1**
1840	6.89	−1.86	− .13	3
1850	6.46	− .43	− .56	4
1860	4.66	−1.80	−2.36	9
1870	5.63	+ .97	−1.39	5
1880	5.51	− .12	−1.51	6
1890	5.24	− .27	−1.78	7
1900	4.60*	− .64	−2.42	11*
1910	5.06	+ .46	−1.96	8
1920	4.64	− .42	−2.38	10

*Minimum or lowest for the entire period.
**Maximum or highest for the entire period.

Read table as follows: In the decade from 1870 to 1879, exercises in division amounted to 5.63 per cent of the total exercises for the decade. This was an increase of .97 points over the percentage for the decade of the sixties, but a decrease of 1.39 points from that of the twenties. This decade, 1870–79, ranked fifth in the emphasis placed on division. See Fig. 22.

FIG. 22—PERCENTAGE OF EXERCISES IN DIVISION

TABLE 46

DISTRIBUTION OF EXERCISES IN DIVISION ACCORDING TO THE FIFTH IN WHICH THEY APPEARED IN THEIR RESPECTIVE BOOKS

DECADE	PER CENT OF EXERCISES IN DIVISION IN THE				
	FIRST FIFTH	SECOND FIFTH	THIRD FIFTH	FOURTH FIFTH	FIFTH FIFTH
1820	58.52**	28.89	8.15	.00*	4.44
1830	50.34**	34.77	4.70	.00*	10.20
1840	67.97**	18.47	10.00	3.56	.00*
1850	51.49**	39.66	.81	8.04	.00*
1860	42.18	57.03**	.63	.16	.00*
1870	59.49**	39.51	.81	.06*	.12
1880	58.70**	38.19	2.82	.04*	.25
1890	27.52	68.00**	4.26	.18	.04*
1900	37.71	53.43**	4.24	3.86	.76*
1910	18.30	64.76**	12.22	3.29	1.42*
1920	24.74	53.18**	11.61	7.18	3.29*

*Minimum percentage for the decade.
**Maximum percentage for the decade.

Read table as follows: In the decade beginning in 1820, 58.52 per cent of the exercises in division appeared in the first fifth of their respective books, and 28.89 per cent in the second fifth. The third, fourth, and last fifths had, respectively, 8.15, 0.00, and 4.44 per cent of the exercises in this topic.

The percentages given in this table have been computed according to the method described on page ix–x.

TABLE 47

PERCENTAGE OF SIMPLE AND COMPLICATED EXERCISES IN DIVISION FOR EACH DECADE

DECADE	PER CENT OF EXERCISES	
	SIMPLE	COMPLICATED
1820	58.89	41.11
1830	72.61	27.39
1840	61.97	38.03
1850	63.09	36.91
1860	52.95	47.05
1870	65.76	34.24
1880	44.99*	55.01**
1890	51.85	48.15
1900	51.88	48.12
1910	69.65	30.35
1920	80.81**	19.19*

*Minimum percentage for the entire period.
**Maximum percentage for the entire period.

Read table as follows: In the first decade, 58.89 per cent of the exercises in division were simple exercises and 41.11 per cent were complicated.

The types of exercises in division that compose the two groups—simple and complicated—are listed on page 84. See Fig. 23.

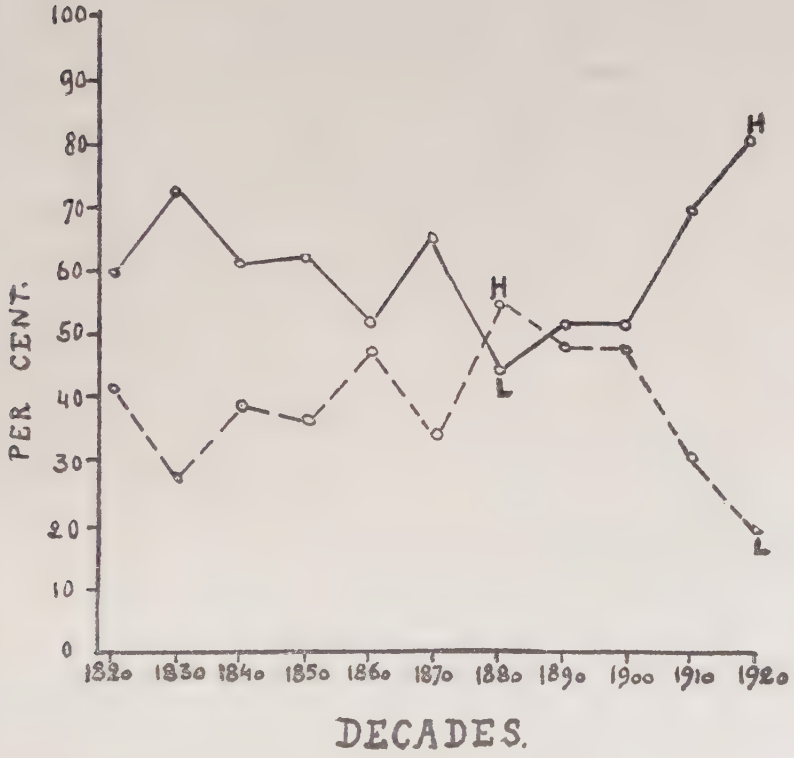

FIG. 23—COMPARISON OF SIMPLE AND COMPLICATED EXERCISES IN DIVISION
_____ Represents simple exercises in division.
_ _ _ _ Represents complicated exercises in division.

TABLE 48

PERCENTAGE OF EXERCISES IN DIVISION DEVOTED TO SHORT DIVISION, LONG DIVISION, AND SPECIAL QUOTIENTS, RESPECTIVELY

DECADE	PER CENT OF EXERCISES IN DIVISION DEVOTED TO		
	SHORT DIVISION	LONG DIVISION	SPECIAL QUOTIENTS
1820	60.74	39.26	.00*
1830	68.62**	31.38	.00*
1840	60.96	38.58	.46
1850	60.40	39.60	.00*
1860	50.79	39.57**	9.65
1870	62.81	32.22	4.97
1880	43.69	36.33	19.98
1890	41.62*	37.03	21.35**
1900	49.23	37.98	12.80
1910	59.46	29.76*	10.78
1920	65.58	30.22	4.20

*Minimum percentage for the entire period.
**Maximum percentage for the entire period.

Read table as follows: In the first decade, 60.74 per cent of the exercises in division were in short division and 39.26 per cent were in long division. There were no exercises in special quotients. See Fig. 24.

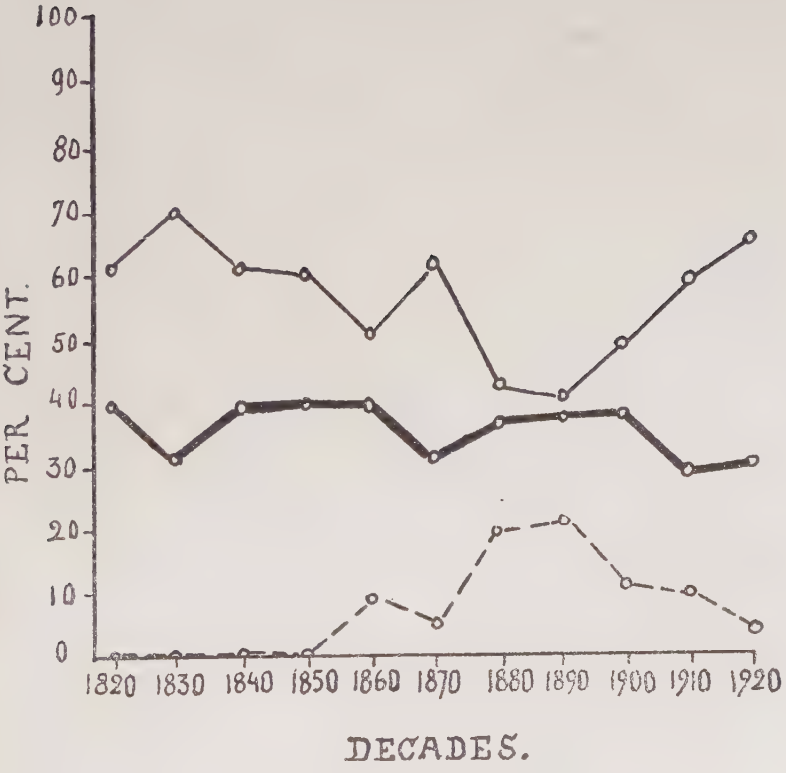

FIG. 24—COMPARISON OF SHORT DIVISION, LONG DIVISION, AND SPECIAL QUOTIENTS

_____ Represents exercises in short division.
_____ Represents exercises in long division.
_ _ _ Represents exercises in special quotients.

Discussion

Table 44 shows that the mean number of exercises in division has increased from 27.0 in the first decade to 165.6 in the present decade. Only one decade, 1900–09, decreased in mean number from its preceding decade. This decrease, however, amounted to only 11.3 exercises or 8.3 per cent. The largest percentage of increase occurred in the thirties, when the mean number of exercises almost doubled.

The percentage of exercises in division is shown for each decade in Table 45. In the first decade, the percentage amounted to 7.02. This increased to 8.75 per cent in the next decade. From then on, with the exception of the seventies, it gradually decreased, the minimum being reached in the first decade of the twentieth century. Since then the percentage, amounting in the present decade to 4.64 per cent, has varied less than one point. Figure 22 shows these changes graphically.

As the data in Table 46 show, division on the whole has followed the same general trend as multiplication in the distribution of its exercises. However, the tendency to postpone the largest percentage of drill to the second fifth started earlier in the case of division. More than 50 per cent of the exercises in this topic have been in the second fifth since the decade of the nineties. The slight percentages in the remaining fifths are mostly due to drill with literal exponents.

It is evident from Table 47 and Figure 23 that in the decades between 1830 and 1859 the simple exercises in division exceeded the complicated by a wide margin, the difference in percentage generally being from 20 to 40 points. However, in the first decade and in the decades of the sixties, eighties, and nineties, the percentage of simple exercises was much smaller. In 1880–89 less than half the exercises in division (44.99) were simple. Complicated exercises consequently made up more than half the drill in this topic in this decade. In the following two decades, they amounted to slightly over 48 per cent. In the decade from 1910 to 1919, however, the percentage decreased greatly. The tendency to decrease has continued in the present decade, with the result that complicated exercises now total only 19.19 per cent. Hence drill in division is much simpler now. In this

respect division in the present decade differs especially from the last decades of the nineteenth century.

Table 48 and Figure 24 show that short division comprised over 60 per cent of the exercises in division in each of the first four decades of the period. In the third decade (1840–49), the percentage began to decrease. It continued to decrease with the exception of the decade from 1870 to 1879. The minimum (41.62 per cent) was reached in the last decade of the century. As in many other topics, the decade of the seventies did not follow the tendency of the times but reverted to the percentage usual in the earlier decades of the century. Thus, exercises in short division increased to 62.81 per cent in that decade. Since the beginning of the twentieth century the percentage has increased. In the present decade, exercises in this kind of division comprise 65.58 per cent of the exercises in this topic. This is the second highest percentage for the entire period, the maximum being 68.62 per cent in the decade from 1830 to 1839.

Long division has constituted throughout the period from 29.76 to 39.57 per cent of the drill in division. The minimum percentage occurred in 1910–19, and the maximum in 1850–59. Long division in the present decade comprises 30.22 per cent of the exercises. This is the second lowest percentage in it for the period. No consistent tendency towards increase or decrease is noticeable.

Special quotients received little or no drill until the decade of the sixties. In 1860–69, drill in them amounted to 9.65 per cent. In the following decade it dropped to 4.97 per cent, but in the eighties it increased to 19.98 per cent. This was approximately double the percentage in the sixties and four times the percentage in the seventies. The maximum, 21.35 per cent, was reached in the last decade of the nineteenth century. Since the beginning of the twentieth century, there has been a large decrease. As a result, only 4.20 per cent of the exercises are in special quotients in the present decade. This percentage differs greatly from those in the last three decades of the past century, but is practically the same as the percentage for the decade from 1870 to 1879.

Summary

There are in the present decade more exercises in division than previously, although the relative emphasis placed on division is less than in the earliest decades of the period. More drill comes later in the course now. In the present decade the exercises are more similar in composition to those in the seventies and the earlier decades of the last century than to those in the decades from 1880 to 1909 inclusive. More are simple exercises in the present decade than in the past.

IX

Proportions and Progressions

Proportions and progressions form the subject of this chapter. Included in it are geometric and arithmetic ratios, proportions, and progressions. The discussion is preceded by the tables and charts which are briefly described in the following paragraphs.

Table 49 gives the mean number of exercises, together with a comparison of the increase or decrease in each decade. Table 50 gives the percentage of exercises in proportions and progressions, also the changes in these percentages from decade to decade and from 1820–29. The distribution of the exercises according to the fifth in which they appeared in the textbooks is shown in Table 51.

TABLE 49

Mean Number of Exercises in Proportions and Progressions per Book, Together With a Comparison of the Increase or Decrease in Each Decade

Decade	Mean number of exercises	Change from preceding decade		
		Number of exercises	Per cent increase or decrease	Rank in percentage of increase
1820	12.9*			
1830	33.8	+20.9	+162.0	1**
1840	54.3	+20.5	+ 60.6	2
1850	77.0	+22.7	+ 41.8	5
1860	114.8	+37.8	+ 49.4	4
1870	152.8	+38.0	+ 33.1	6
1880	163.2**	+10.4	+ 6.8	7
1890	134.3	−28.9	− 17.7	9
1900	102.7	−31.6	− 25.3	10*
1910	97.5	− 5.2	− 5.1	8
1920	149.3	+51.8	+ 53.1	3

*Minimum or lowest for the entire period.
**Maximum or highest for the entire period.

Read table as follows: In the decade from 1830 to 1839, the mean number of exercises in proportions and progressions amounted to 33.8 exercises per book, an increase of 20.9 exercises or 162.0 per cent over the decade of the twenties. The decade of the thirties ranked first in the percentage of increase in mean number of exercises in proportions and progressions.

The exercises in infinite geometric progression were considered most complicated and are compared with the remainder of the exercises in Table 52.

The percentages of exercises in proportions and progressions which involve respectively geometric ratio and proportion, arithmetic ratio and proportion, geometric progression, and arithmetic progression are compared in Table 53.

TABLE 50

PERCENTAGE OF EXERCISES IN PROPORTIONS AND PROGRESSIONS, TOGETHER WITH A COMPARISON OF THE INCREASE OR DECREASE IN EACH DECADE

DECADE	EXERCISES IN PROPORTIONS AND PROGRESSIONS			
	PER CENT	CHANGE FROM PRECEDING DECADE	CHANGE FROM 1820	RANK IN PERCENTAGE OF EXERCISES
1820	3.36			10
1830	5.56	+2.20	+2.20	6
1840	6.34	+ .78	+2.98	5
1850	8.07	+1.73	+4.71	3
1860	8.88	+ .81	+5.52	2
1870	9.13**	+ .25	+5.77	1**
1880	7.47	−1.66	+4.11	4
1890	5.13	−2.34	+1.77	7
1900	3.75	−1.38	+ .39	9
1910	3.11*	− .64	− .25	11*
1920	4.18	+1.07	+ .82	8

*Minimum or lowest for the entire period.
**Maximum or highest for the entire period.

Read table as follows: In the decade from 1880 to 1889, exercises in proportions and progressions amounted to 7.47 per cent of the total exercises for the decade. This was a decrease of 1.66 points from the percentage for the seventies, but an increase of 4.11 points over that for the twenties. The decade of the eighties ranked fourth in emphasis placed on proportions and progressions. See Fig. 25.

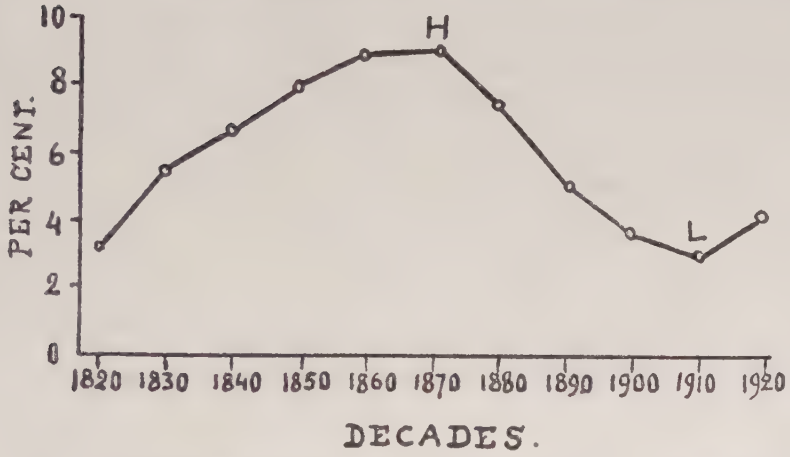

FIG. 25—PERCENTAGE OF EXERCISES IN PROPORTIONS AND PROGRESSIONS

TABLE 51

DISTRIBUTION OF EXERCISES IN PROPORTIONS AND PROGRESSIONS ACCORDING TO THE FIFTH IN WHICH THEY APPEARED IN THEIR RESPECTIVE BOOKS

DECADE	PER CENT OF EXERCISES IN PROPORTIONS AND PROGRESSIONS IN THE				
	FIRST FIFTH	SECOND FIFTH	THIRD FIFTH	FOURTH FIFTH	FIFTH FIFTH
1820	.00*	.00*	17.85	42.64**	39.53
1830	.00*	.85	8.88	15.86	74.42**
1840	.09*	9.86	1.48	2.95	85.62**
1850	.00*	3.90	7.85	16.45	71.81**
1860	.77	1.45	24.88	.00*	72.89**
1870	.00*	2.04	13.36	13.36	71.25**
1880	.00*	1.13	10.82	3.06	84.98**
1890	.00*	.00*	.00*	5.05	94.95**
1900	.02*	.02*	5.11	7.13	87.71**
1910	4.40*	8.80	8.01	29.14	49.65**
1920	5.48	1.21*	17.50	31.68	44.14**

*Minimum percentage for the decade.
**Maximum percentage for the decade.

Read table as follows: In the first decade, 17.85 per cent of the exercises in proportions and progressions appeared in the third fifth of their respective books; 42.64 per cent appeared in the fourth fifth; and 39.53 per cent appeared in the last fifth. None appeared in the first two fifths.

The percentages given in this table have been computed according to the method described on page ix–x.

TABLE 52

PERCENTAGE OF SIMPLE AND COMPLICATED EXERCISES IN PROPORTIONS AND PROGRESSIONS FOR EACH DECADE

DECADE	PER CENT OF EXERCISES	
	SIMPLE	COMPLICATED
1820	96.90	3.10
1830	96.76	3.24
1840	91.98*	8.02**
1850	93.30	6.70
1860	92.81	7.19
1870	95.76	4.24
1880	92.97	7.03
1890	94.46	5.54
1900	94.88	5.12
1910	100.00**	0.00*
1920	100.00**	0.00*

*Minimum percentage for the entire period.
**Maximum percentage for the entire period.

Read table as follows: In the first decade, 96.90 per cent of the exercises in proportions and progressions were simple exercises, and 3.10 per cent were complicated. See Fig. 26.

The types of exercises in proportions and progressions that compose the two groups—simple and complicated—are listed on page 94.

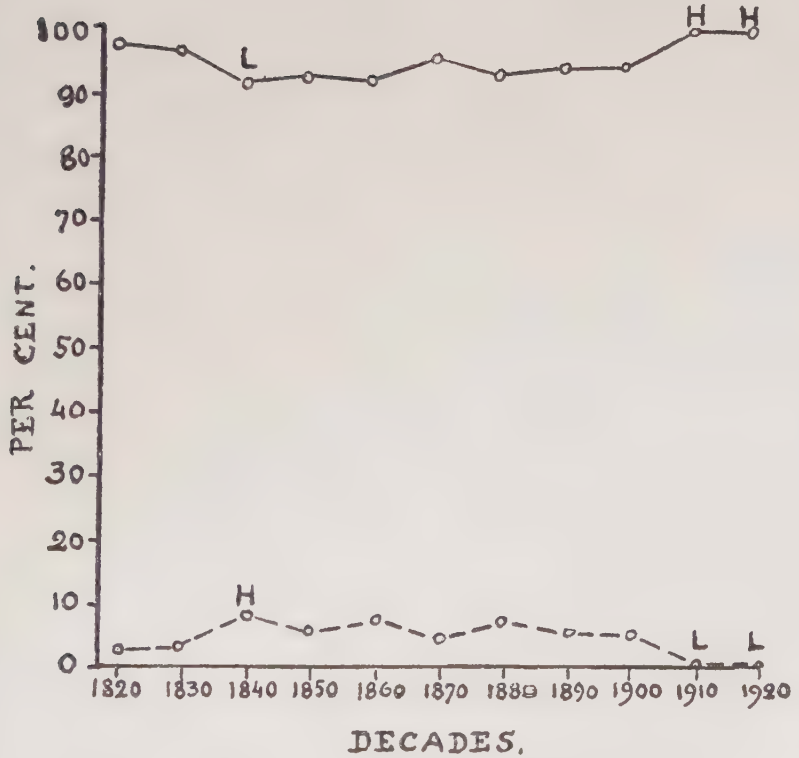

FIG. 26—COMPARISON OF SIMPLE AND COMPLICATED EXERCISES IN PROPORTIONS AND PROGRESSIONS

———— Represents simple exercises in proportions and progressions.
— — — Represents complicated exercises in proportions and progressions.

TABLE 53

PERCENTAGE OF EXERCISES IN PROPORTIONS AND PROGRESSIONS DEVOTED TO GEOMETRIC RATIO AND PROPORTION, ARITHMETIC RATIO AND PROPORTION, GEOMETRIC PROGRESSION, AND ARITHMETIC PROGRESSION, RESPECTIVELY

DECADE	PER CENT OF PROPORTIONS AND PROGRESSIONS DEVOTED TO			
	RATIO AND PROPORTION		PROGRESSIONS	
	GEOMETRIC	ARITHMETIC	GEOMETRIC	ARITHMETIC
1820	23.26*	5.43**	21.71	49.61**
1830	34.63	.00*	24.27	41.10
1840	24.73	1.47	32.75	41.04
1850	27.20	.33	35.02**	37.45
1860	36.38	.41	28.49	34.72
1870	39.91	.00*	26.89	33.20
1880	36.56	.00*	30.40	33.04
1890	34.76	.00*	30.21	35.03
1900	45.06	.00*	27.05	27.89
1910	100.00**	.00*	.00*	.00*
1920	100.00**[1]	.00*	.00*	.00*

*Minimum percentage for the entire period.
**Maximum percentage for the entire period.
[1] Trigonometric ratios make up 30.29 per cent of this.

Read table as follows: In the decade beginning in 1820, 23.26 per cent of the exercises in proportions and progressions dealt with geometric ratio and proportion; 5.43 per cent dealt with arithmetic ratio and proportion; 21.71 per cent with geometric progression; and 49.61 per cent with arithmetic progression. See Fig. 27.

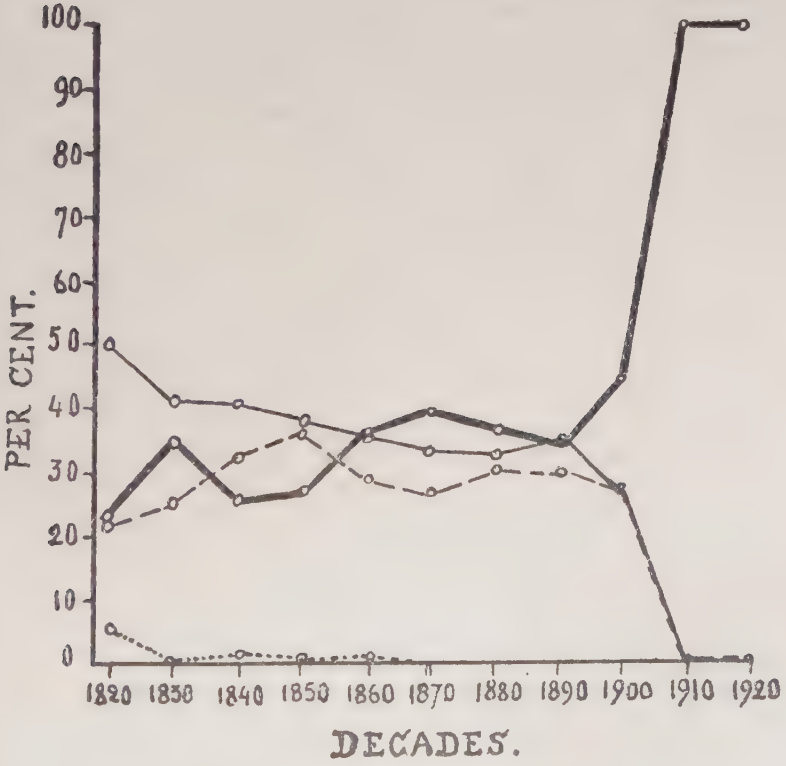

FIG. 27—COMPARISON OF GEOMETRIC AND ARITHMETIC PROPORTIONS AND PROGRESSIONS

——— Represents exercises in geometric ratio and proportion.
. . . Represents exercises in arithmetic ratio and proportion.
_ _ _ Represents exercises in geometric progression.
_ . _ Represents exercises in arithmetic progression.

Discussion

Table 49 shows that the mean number of exercises in proportions and progressions increased from 12.9 exercises in the first decade to 163.2 in the decade of the eighties. In the nineties it decreased to 134.3. This was 28.9 exercises less than in the preceding decade and represented a loss of 17.7 per cent. The tendency to decrease continued until the present decade was reached. The mean number, amounting to 97.5 exercises in the decade beginning in 1910, has increased to 149.3 exercises in the present decade. This is a gain of 51.8 exercises or 53.1 per cent over 1910–19.

As Table 50 and Figure 25 show, the percentage of exercises in proportions and progressions followed the same trend as the mean number of exercises with the exception that the percentage began to decrease in the decade of the eighties, about ten years sooner than in the case of the mean number of exercises. In the first decade, proportions and progressions amounted to 3.36 per cent. The percentage increased in the following decades, the maximum amounting to 9.13 per cent in 1870–79. In 1880–89, the percentage decreased to 7.47. It decreased in 1890–99 to 5.13, in 1900–09 to 3.75, and in 1910–19 to 3.11. In the present decade, however, this topic has increased to 4.18 per cent. This is due primarily to exercises in similar triangles and trigonometric ratios. The maximum percentage occurred in 1870–79 and the minimum in 1910–19.

As is shown in Table 51, the largest percentage of exercises in proportions and progressions has been in the last fifth with the exception of the first decade, in which it was in the fourth fifth. In the nineteenth century, every decade after the first had from 71.25 to 94.95 per cent of the exercises of this topic in the last fifth. In 1900–09, 87.71 per cent were in this fifth, but in 1910–19 there was a large decrease. The tendency to decrease has continued in the present decade, only 44.14 per cent now being in the last fifth. This indicates a tendency to distribute in the earlier fifths some of the drill formerly given very late in the course. The present decade has 5.48 per cent of the exercises in this topic in the first fifth; the decade from 1910 to 1919 has

4.40 in this fifth. Previously the largest percentage in the first fifth amounted to .77, found in 1860–69. Thus, the first fifth has increased in percentage of exercises. The second fifth, however, has not shown a clear tendency in this direction.

It is evident from Table 53 that since the decade of the sixties larger percentages than previously have been the rule in proportions (arithmetic and geometric, combined). Preceding the sixties, proportions varied between 26.20 and 34.63 per cent. In the decades between 1860 and 1899, however, they ranged from 34.76 to 39.91 per cent. In the first decade of the twentieth century, the percentage increased to 45.06. In the last two decades it has been 100 per cent.

Although progressions exceeded proportions in percentage of exercises until the 1910 decade, the tendency on the whole has been for the former to decrease. Preceding about 1860, progressions ranged from 65.37 to 73.79 per cent. In the remainder of the past century they varied between 60.09 and 65.24 per cent. In 1900–09, progressions amounted to 54.94 per cent of the exercises in proportions and progressions, but since 1910–19 they have been entirely omitted.

By comparing the graphs for arithmetic and geometric proportions in Figure 27, it may be seen that the latter always received the greater amount of drill. In fact, arithmetic proportion has not received any drill since the sixties. Previous to that decade, it usually received a small amount, the maximum being 5.43 per cent in the first decade of the period. The minimum percentage in geometric proportion occurred in the decade from 1820 to 29, the maxima in the decades beginning in 1910 and in 1920. The tendency, on the whole, has been for its percentage to increase. This may be seen by comparing the range of the percentages preceding the sixties with those from 1860 to 1899, and with those in the present century. In the last two decades, the drill in geometric proportion has amounted to 100 per cent. In the present decade, 30.19 per cent of this amount is devoted to similar triangles and trigonometric ratios. Exercises in the latter did not become a part of the elementary course until the present decade, as judged by the books examined.

Arithmetic progression received more drill than geometric

progression as long as both formed part of the course. However, arithmetic progression decreased from decade to decade with the exception of the last decade of the nineteenth century. In the first decade of the period, drill in it amounted to 49.61 per cent. In the first decade of the twentieth century it totalled 27.89 per cent, a difference of 21.72 points from the percentage for the decade, 1820–29. Geometric progression, on the other hand, although having smaller percentages of exercises than arithmetic progression, increased preceding the sixties and again in the last twenty years of the century. The largest percentage occurred in the middle of the past century (1850–59). In that decade 35.02 per cent dealt with geometric progression. The minimum, preceding the total elimination of progressions from the course, came in 1820–29 and amounted to 21.71 per cent.

Summary

Proportions and progressions received the most emphasis in the middle decades of the past century. Both the percentage and the mean number of exercises have decreased since the decades of the seventies and eighties. The present decade, however, shows an increase in both percentage and mean number, the first to 4.18 per cent and the second to 149.3 exercises. The relative distribution of drill among the various phases of this topic has changed greatly. Work in progressions, which comprised more than half the drill until the two most recent decades, has been entirely eliminated since. Arithmetic proportion was eliminated about the middle of the century. Geometric proportion, the only one of these four phases left, now includes a large percentage (30.19) of exercises in similar triangles and trigonometric ratios.

X

Negative Exponents

In the first part of this chapter are given the tables and charts showing the development of negative exponents in mean number, percentage, and distribution of exercises. This is followed by the discussion based on them.

TABLE 54

Mean Number of Exercises in Negative Exponents per Book, Together With a Comparison of the Increase or Decrease in Each Decade

Decade	Mean number of exercises	Change from preceding decade		
		Number of exercises	Per cent increase or decrease	Rank in percentage of increase
1820	4.3			
1830	1.6*	− 2.7	− 62.8	10*
1840	7.1	+ 5.5	+343.7	1**
1850	8.9	+ 1.8	+ 25.3	5
1860	21.6	+12.7	+142.7	2
1870	22.4	+ .8	+ 3.7	7
1880	38.4	+16.0	+ 71.4	3
1890	41.8	+ 3.4	+ 8.9	6
1900	42.2**	+ .4	+ 1.0	8
1910	17.3	−24.9	− 59.0	9
1920	21.9	+ 4.6	+ 26.7	4

*Minimum or lowest for the entire period.
**Maximum or highest for the entire period.
Read table as follows: In the decade from 1830 to 1839, the mean number of exercises in negative exponents amounted to 1.6 exercises per book, a decrease of 2.7 exercises or 62.8 per cent from the preceding decade, 1820–29. The decade of the thirties ranked tenth in the percentage of increase in the mean number of exercises in negative exponents.

Table 54, in addition to giving the mean number of exercises, compares the increase or decrease in each decade with the preceding decade. The percentage of exercises in negative exponents and the change in these percentages from decade to decade and from the first decade is given in Table 55.

The distribution of the exercises in negative exponents accord-

ing to the fifth in which they appeared in their respective books is indicated in Table 56.

TABLE 55

PERCENTAGE OF EXERCISES IN NEGATIVE EXPONENTS, TOGETHER WITH A COMPARISON OF THE INCREASE OR DECREASE IN EACH DECADE

DECADE	EXERCISES IN NEGATIVE EXPONENTS			
	PER CENT	CHANGE FROM PRECEDING DECADE	CHANGE FROM 1820	RANK IN PERCENTAGE OF EXERCISES
1820	1.12			6
1830	.27*	− .85	− .85	11*
1840	.83	+ .56	− .29	8
1850	.93	+ .10	− .19	7
1860	1.40	+ .47	+ .28	4
1870	1.34	− .06	+ .22	5
1880	1.76**	+ .42	+ .64	1**
1890	1.59	− .17	+ .47	2
1900	1.54	− .05	+ .42	3
1910	.55	− .99	− .57	10
1920	.61	+ .06	− .51	9

*Minimum or lowest for the entire period.
**Maximum or highest for the entire period.

Read table as follows: In the decade of the nineties, exercises in negative exponents amounted to 1.59 per cent of the total exercises for the decade. This was a decrease of .17 points from the percentage for the eighties, but an increase of .47 points over that for the twenties. This decade, 1890–99, ranked second in the emphasis placed on negative exponents.

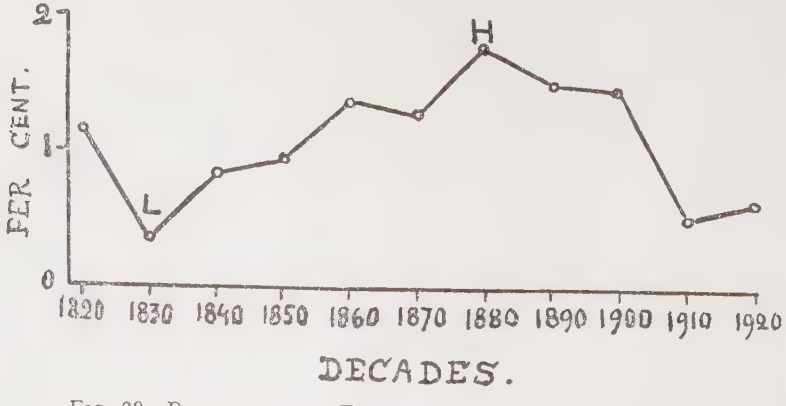

FIG. 28—PERCENTAGE OF EXERCISES IN NEGATIVE EXPONENTS

TABLE 56

DISTRIBUTION OF EXERCISES IN NEGATIVE EXPONENTS ACCORDING TO THE FIFTH IN WHICH THEY APPEARED IN THEIR RESPECTIVE BOOKS

DECADE	PER CENT OF EXERCISES IN NEGATIVE EXPONENTS IN THE				
	FIRST FIFTH	SECOND FIFTH	THIRD FIFTH	FOURTH FIFTH	FIFTH FIFTH
1820	.00*	.00*	100.00**	.00*	.00*
1830	.00*	8.70	73.91**	.00*	17.39
1840	26.76	4.93	39.44**	.70*	28.17
1850	39.25	40.65**	13.55	2.34*	4.21
1860	2.56	36.41**	33.85	27.18	.00*
1870	5.25*	32.54**	29.13	12.60	20.47
1880	6.25*	13.15	11.20	49.61**	19.79
1890	11.97	7.26	.73*	45.43**	34.62
1900	4.42	5.17	.97*	50.19**	39.26
1910	.00*	7.82	.00*	18.25	73.93**
1920	.00*	.00*	7.82	16.04	76.14**

*Minimum percentage for the decade.
**Maximum percentage for the decade.

Read table as follows: In the decade beginning in 1840, 26.76 per cent of the exercises in negative exponents appeared in the first fifth of their respective books; 4.93 per cent appeared in the second fifth; 39.44 per cent, in the third fifth; .70 per cent, in the fourth fifth; and 28.17 per cent, in the last fifth.

The percentages in this table have been computed according to the method described on page ix-x.

Discussion

Table 54 shows that the mean number of exercises in negative exponents amounted to 4.3 exercises at the beginning of the period. This decreased to 1.6 exercises in the decade of the thirties. From the decade beginning in 1840 to the first decade of the twentieth century, the mean number increased steadily, amounting to 42.2 exercises in 1900–09. In the following decade, however, the number decreased to less than half of this—17.3. In the present decade it consists of 21.9 exercises, which is higher than in 1910–19, but only slightly more than half the number for 1900–09. The mean number now is approximately the same as in the decades beginning in 1860 and in 1870.

Table 55 and Figure 28 show that the percentage of exercises in negative exponents has been less than two per cent throughout the period. In the first decade, exercises in this topic amounted

to 1.12 per cent of the exercises for the decade. This decreased to .27 (the minimum) in the second decade. From about 1840 on, however, they increased slowly in percentage, the maximum of 1.76 per cent being reached in 1880–89. Since then, however, the percentage has generally decreased and in the present decade amounts to only .61.

In the first decade, Table 56 shows, negative exponents came in the middle of the course, *i.e.*, in the third fifth. In the second decade two opposing tendencies began, the one to postpone the drill until later, the other to introduce it earlier. Both tendencies became stronger in the forties, but neither was strong enough to draw the largest percentage of exercises from the third fifth. In the decade from 1850 to 1859, however, the tendency towards early drill became so pronounced that 79.90 per cent of the exercises came in the first two-fifths of the books. The largest percentage, 40.65, came in the second fifth. The maximum continued in this fifth in the sixties and seventies but in each case the percentage became smaller. In 1880–89 the largest percentage shifted to the fourth fifth and in 1910–19, to the last fifth. Since then, it has remained in this fifth. In the present decade there is no drill in this topic in the first two fifths, and only 7.82 per cent in the third fifth. Thus the tendency, which first appeared in 1830–39, to postpone the drill in negative exponents to the latter part of the course, reappeared later in the century and has since continued to the present.

Summary

The topic of negative exponents has been, throughout the period, one of the least important in elementary algebra. Drill in it has always been slight and in the present decade is found mostly in the last part of the course.

XI

Parentheses

The purpose of this chapter is to show the development of parentheses since the beginning of the high school movement, as revealed by the textbooks examined. The discussion given is based on the data presented in the tables and charts that precede it. Table 57 shows the mean number of exercises, together with a comparison of the increase or decrease in each decade. Table 58 indicates the percentage of exercises and the changes in these percentages from the first decade and from decade to decade. The distribution of exercises in parentheses according to the fifth in which they appeared in their respective books is given in Table 59.

TABLE 57

Mean Number of Exercises in Parentheses Per Book, Together With a Comparison of the Increase or Decrease in Each Decade

Decade	Mean number of exercises	Change from preceding decade		
		Number of exercises	Per cent increase or decrease	Rank in percentage of increase
1820	.9*			
1830	2.1	+ 1.2	+133.9	2
1840	4.8	+ 2.7	+128.6	3
1850	6.8	+ 2.0	+ 41.7	6
1860	19.0	+12.2	+179.4	1**
1870	21.8	+ 2.8	+ 14.8	8
1880	36.9	+15.1	+ 69.3	4
1890	50.5	+13.6	+ 36.9	7
1900	44.8	− 5.7	− 11.3	10*
1910	68.5	+23.7	+ 52.9	5
1920	69.9**	+ 1.4	+ 2.0	9

*Minimum or lowest for the entire period.
**Maximum or highest for the entire period.

Read table as follows: In the decade from 1830 to 1839, the mean number of exercises in parentheses amounted to 2.1 exercises per book, an increase of 1.2 exercises or 133.9 per cent over the decade of the twenties. The decade of the thirties ranked second in percentage of increase in the mean number of exercises in parentheses.

Table 60 contains a comparison of the simple and complicated exercises in this topic. Exercises in removal of "nests", *i.e.*, exercises containing more than two parentheses within each other, and exercises in the insertion of parentheses within each other compose the complicated group. All other exercises in this topic have been considered simple exercises.

Exercises in the removal of parentheses are compared with exercises in the insertion of parentheses in Table 61, the percentage of each being given.

TABLE 58

PERCENTAGE OF EXERCISES IN PARENTHESES, TOGETHER WITH A COMPARISON OF THE INCREASE OR DECREASE IN EACH DECADE

DECADE	EXERCISES IN PARENTHESES			
	PER CENT	CHANGE FROM PRECEDING DECADE	CHANGE FROM 1820	RANK IN PERCENTAGE OF EXERCISES
1820	.23*			11*
1830	.34	+ .11	+ .11	10
1840	.55	+ .21	+ .32	9
1850	.71	+ .16	+ .48	8
1860	1.26	+ .55	+1.03	7
1870	1.30	+ .04	+1.07	6
1880	1.69	+ .39	+1.46	4
1890	1.93	+ .24	+1.70	3
1900	1.64	− .29	+1.41	5
1910	2.18**	+ .54	+1.95	1**
1920	1.96	− .22	+1.73	2

*Minimum or lowest for the entire period.
**Maximum or highest for the entire period.

Read table as follows: In the decade from 1900 to 1909, exercises in parentheses amounted to 1.64 per cent of the total exercises for the decade. This was a decrease of .29 points from the percentage for the nineties, but an increase of 1.41 points over that for the twenties. This decade ranked fifth in the emphasis placed on parentheses. See Fig. 29.

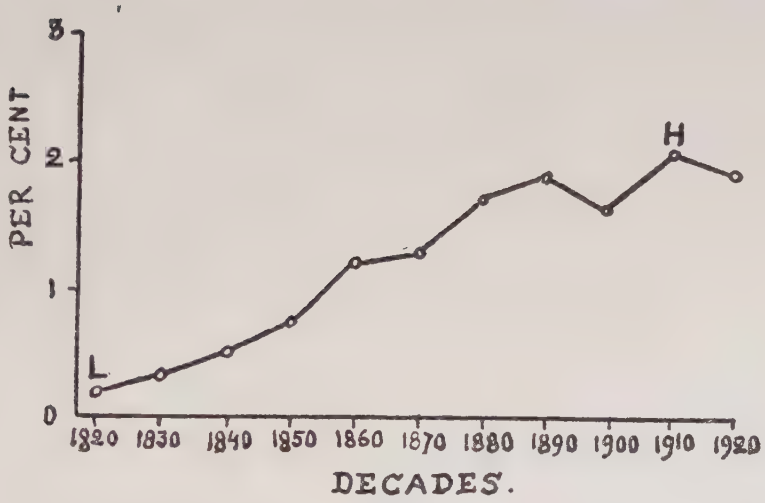

FIG. 29—PERCENTAGE OF EXERCISES IN PARENTHESES

TABLE 59

DISTRIBUTION OF EXERCISES IN PARENTHESES ACCORDING TO THE FIFTH IN WHICH THEY APPEARED IN THEIR RESPECTIVE BOOKS

DECADE	PER CENT OF EXERCISES IN PARENTHESES IN THE				
	FIRST FIFTH	SECOND FIFTH	THIRD FIFTH	FOURTH FIFTH	FIFTH FIFTH
1820	100.00**	.00*	.00*	.00*	.00*
1830	75.86**	.00*	.00*	24.14	.00*
1840	72.63**	27.37	.00*	.00*	.00*
1850	79.75**	9.82	10.43	.00*	.00*
1860	90.64**	9.36	.00*	.00*	.00*
1870	76.22**	22.43	1.35	.00*	.00*
1880	86.31**	12.60	1.08	.00*	.00*
1890	72.31**	27.15	.55	.00*	.00*
1900	88.70**	10.49	.81	.00*	.00*
1910	43.12	50.94**	2.64	3.30	.00*
1920	29.90	60.16**	2.74	7.20	.00*

*Minimum percentage for the decade.
**Maximum percentage for the decade.

Read the table as follows: In the decade beginning in 1850, 79.75 per cent of the exercises in parentheses appeared in the first fifth of their respective books; 9.82 per cent appeared in the second fifth; and 10.43 per cent, in the third fifth. None appeared in the last two fifths.

The percentages in this table have been computed according to the method described on page ix–x.

TABLE 60

PERCENTAGE OF SIMPLE AND COMPLICATED EXERCISES IN PARENTHESES FOR EACH DECADE

DECADE	PER CENT OF EXERCISES	
	SIMPLE	COMPLICATED
1820	100.00**	0.00*
1830	100.00**	0.00*
1840	100.00**	0.00*
1850	97.66	2.34
1860	96.83	3.17
1870	94.47	5.53
1880	83.79	16.21
1890	84.86	15.14
1900	83.71*	16.28**
1910	93.59	6.41
1920	98.45	1.55

*Minimum percentage for the entire period.
**Maximum percentage for the enirre period.

Read table as follows: In the decades beginning in 1820, 1830, and 1840 all the exercises in parentheses were simple exercises; that is they contained no "nests". In 1850–59, 97.66 per cent contained no nests and 2.34 per cent did. See Fig. 30.

The types of exercises in parentheses that compose the two groups—simple and complicated—are listed on page 108.

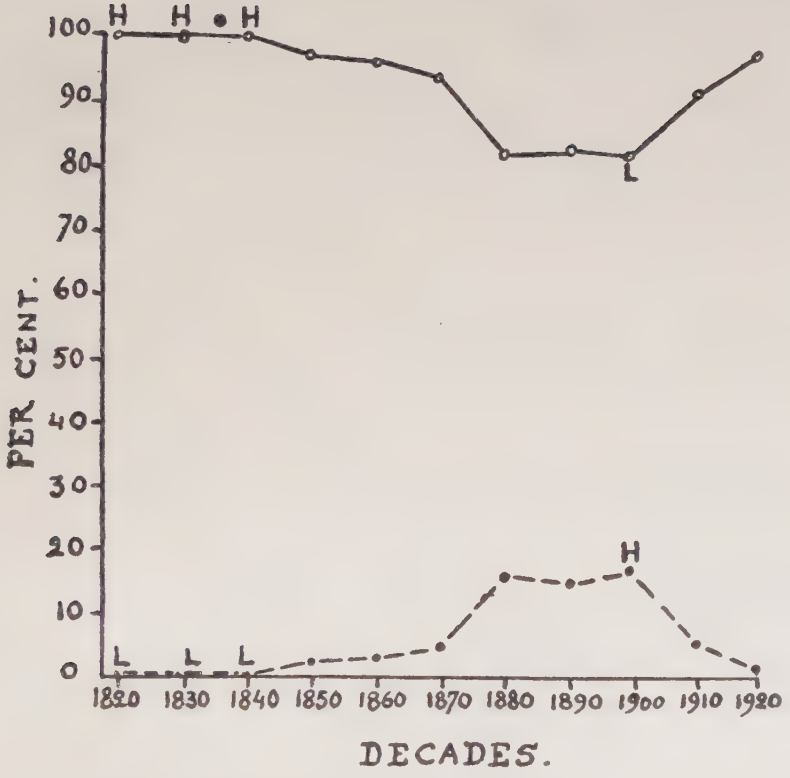

FIG. 30—COMPARISON OF SIMPLE AND COMPLICATED EXERCISES IN PARENTHESES

——— Represents simple exercises in parentheses.
— — — Represents complicated exercises in parentheses.

TABLE 61

PERCENTAGE OF EXERCISES IN PARENTHESES DEVOTED TO THE REMOVAL OF PARENTHESES AND THE INSERTION OF PARENTHESES, RESPECTIVELY

DECADE	PER CENT OF EXERCISES IN PARENTHESES DEVOTED TO	
	REMOVAL OF PARENTHESES	INSERTION OF PARENTHESES
1820	100.00**	00.00*
1830	65.22*	34.78**
1840	68.97	31.03
1850	79.69	20.31
1860	70.63	29.37
1870	80.85	19.15
1880	67.73	32.27
1890	74.48	25.52
1900	75.09	24.91
1910	75.71	24.29
1920	70.14	29.86

*Minimum for the entire period.
**Maximum for the entire period.

Read table as follows: In 1820–29, all the exercises in parentheses were in the removal of parentheses. In 1830–39, 65.22 per cent of the exercises were in removal and 34.78 per cent were in insertion. See Fig. 31.

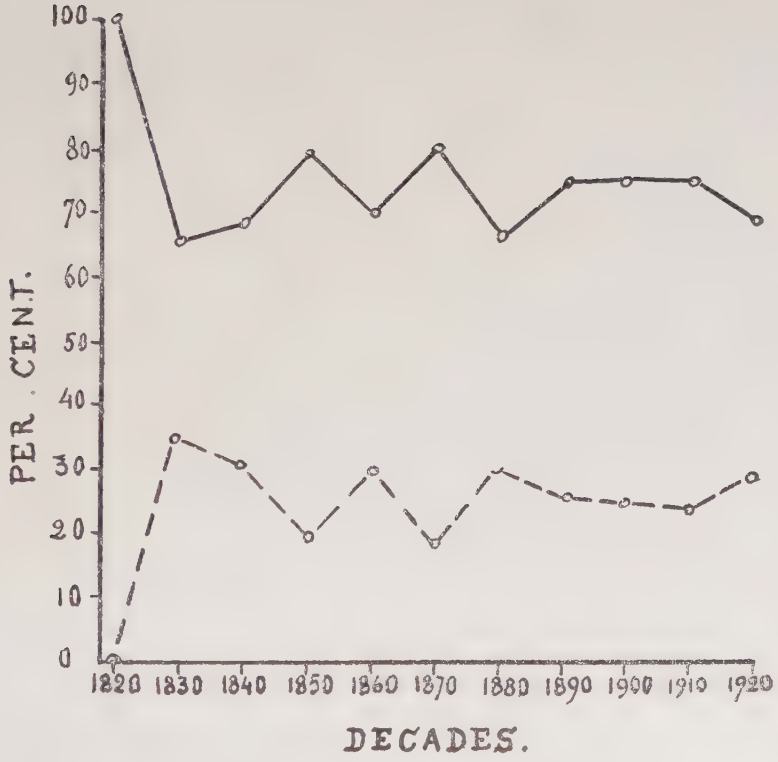

FIG. 31—COMPARISON OF EXERCISES IN THE REMOVAL AND THE INSERTION OF PARENTHESES

_____ Represents exercises in the removal of parentheses.
_ _ _ _ Represents exercises in the insertion of parentheses.

Discussion

Table 57 shows that the mean number of exercises in parentheses amounted to .9 exercises in the first decade. In the second decade, this increased to 2.1. The mean number continued to increase in the following decades. In the sixties it amounted to 19.0 exercises, a gain of 179.4 per cent over the decade of the fifties. By the last decade in the century it had increased to 50.5 exercises. In the following decade (1900–09) it decreased to 44.8 exercises, but in 1910–19 it increased to 68.5. In the present decade the mean number is 69.9. The minimum for the period occurred in the first decade and the maximum, 69.9, in the last or present decade.

As the data in Table 58 show, parentheses exceeded two per cent of the total number of exercises for each decade in only one instance. This was in the decade, 1910–19. In 1820–29, the percentage comprised .23 per cent of the total. Throughout the nineteenth century, parentheses increased slightly in percentage from decade to decade. In the nineties, they amounted to 1.93 per cent. In 1900–09, they decreased to 1.64, but in the following decade increased to 2.18 per cent. In the present decade they amount to 1.96 per cent. As Figure 29 shows, the minimum percentage occurred in 1820–29, and the maximum in 1910–19.

As is indicated in Table 59, most of the exercises in parentheses came in the first fifth of the books until the last two decades. The tendency since 1910 has been to give most of the drill in the second fifth. At present 29.90 per cent are in the first fifth, whereas 60.16 per cent are in the second fifth.

It appears from Table 60 and Figure 30 that drill in nests of parentheses did not begin until the decade of the fifties. In that decade 2.34 per cent of the exercises in parentheses had "nests." This percentage increased only slightly from decade to decade until the decade of the eighties was reached. In that decade the percentage with nests increased to 16.21, three times the percentage for the decade immediately preceding. Between 1880 and 1909, the percentage varied between 15.14 and 16.28 per cent. In the second decade of the twentieth century, however, it decreased to 6.41. It has decreased still further in the present

decade, the percentage amounting to only 1.55 per cent of the total drill in the topic. Thus the percentages of simple exercises in parentheses in the last two decades have been much larger than the percentages in the last decades of the nineteenth century or the first decade of this century. They are more similar to the percentages preceding the sixties than to those following it.

According to Table 61, there was no drill in inserting parentheses until the second decade. In that decade, 34.78 per cent were in insertion. Since the thirties the percentage has varied between 19.15 and 34.78 per cent of the total drill in the topic. No general tendency to increase or decrease is evident.

Summary

Parentheses generally increased in mean number, percentage, and complexity from the beginning of the period studied to the end of the nineteenth century. The mean number is highest in the present decade, but the percentage is slightly lower than in 1910–19. Complicated exercises make up in the present decade a smaller percentage of the exercises than they did a few decades ago.

XII

Factoring, Factors, and Multiples

Factoring, highest common factor, and lowest common multiple are the topics included in this chapter. The discussion of their development is preceded by Tables 62 to 66.

Table 62 shows the mean number of exercises in factoring, factors, and multiples, together with a comparison of the increase or decrease in each decade. The percentage of exercises for each decade and the comparison of the changes in these percentages are given in Table 63. The distribution of the exercises according to the fifths in which they appeared in their respective books is given in Table 64.

Table 65 compares the percentage of simple and complicated exercises in factoring, factors, and multiples. Part I of this table deals with factoring; Part II, with highest common factor and lowest common multiple.

In factoring, the following have been considered simple exercises:

1. Common factors except common compound factors.
2. The difference of two squares.
3. Trinomials of the second degree that can be easily factored by trial. This includes the following types: $(MX^2 \pm PX \pm Q)$; $(A^2 \pm 2AB + B^2)$, $(X^2 \pm PX \pm Q)$.
4. Combinations of 1 with 2 or 3.

The complicated group in factoring is composed of:

1. All expressions with fractional, negative, or literal exponents; radicals or compound terms.
2. The sum or difference of the like powers of two numbers except the difference of two squares.
3. Polynomials necessitating grouping of terms, or the addition and subtraction of a term.
4. Expressions requiring the use of the factor theorem.
5. Miscellaneous cases in factoring.

The simple group in highest common factor and lowest common multiple, comprises the exercises to be solved by factoring; the complicated group consists of those to be solved by division.

Factoring, highest common factor, and lowest common multiple are compared in Table 66, the percentage of exercises in each being given.

TABLE 62

MEAN NUMBER OF EXERCISES IN FACTORING, FACTORS, AND MULTIPLES PER BOOK, TOGETHER WITH A COMPARISON OF THE INCREASE OR DECREASE IN EACH DECADE

DECADE	MEAN NUMBER OF EXERCISES	CHANGE FROM PRECEDING DECADE		
		NUMBER OF EXERCISES	PER CENT INCREASE OR DECREASE	RANK IN PERCENTAGE OF INCREASE
1820	.8*			
1830	5.3	+ 4.5	+562.5	1**
1840	14.3	+ 9.0	+169.8	3
1850	34.3	+ 20.0	+139.9	4
1860	107.0	+ 72.7	+212.0	2
1870	126.9	+ 19.9	+ 18.6	7
1880	252.5	+125.6	+ 99.0	5
1890	359.8	+107.3	+ 42.5	6
1900	368.7	+ 8.9	+ 2.5	8
1910	373.1**	+ 4.4	+ 1.2	9
1920	325.4	− 47.7	− 12.8	10*

*Minimum or lowest for the entire period.
**Maximum or highest for the entire period.

Read table as follows: In the decade beginning in 1830, the mean number of exercises in factoring, factors, and multiples amounted to 5.3 exercises, an increase of 4.5 exercises or 562.5 per cent over the mean number for the twenties. The decade of the thirties ranked first in the percentage of increase in mean number of exercises in factoring, factors, and multiples.

TABLE 63
PERCENTAGE OF EXERCISES IN FACTORING, FACTORS, AND MULTIPLES, TOGETHER WITH A COMPARISON OF THE INCREASE OR DECREASE IN EACH DECADE

DECADE	EXERCISES IN FACTORING, FACTORS, AND MULTIPLES			
	PER CENT	CHANGE FROM PRECEDING DECADE	CHANGE FROM 1820	RANK IN PERCENTAGE OF EXERCISES
1820	.21*			11*
1830	.87	+ .66	+ .66	10
1840	1.66	+ .79	+ 1.45	9
1850	3.59	+1.93	+ 3.38	8
1860	7.09	+3.50	+ 6.88	7
1870	7.58	+ .49	+ 7.37	6
1880	11.56	+3.98	+11.35	4
1890	13.74**	+2.18	+13.53	1**
1900	13.46	− .28	+13.25	2
1910	11.91	−1.55	+11.70	3
1920	9.12	−2.79	+ 8.91	5

*Minimum or lowest for the entire period.
**Maximum or highest for the entire period.

Read table as follows: In the decade from 1910–1919, exercises in factoring, factors, and multiples amounted to 11.91 per cent of the total exercises for the decade. This was a decrease of 1.55 points from the percentage for the preceding decade, 1900–09, but an increase of 11.70 points over that for the decade from 1820 to 1829. This decade, 1910–1919, ranked third in the emphasis placed on factoring, factors, and multiples.

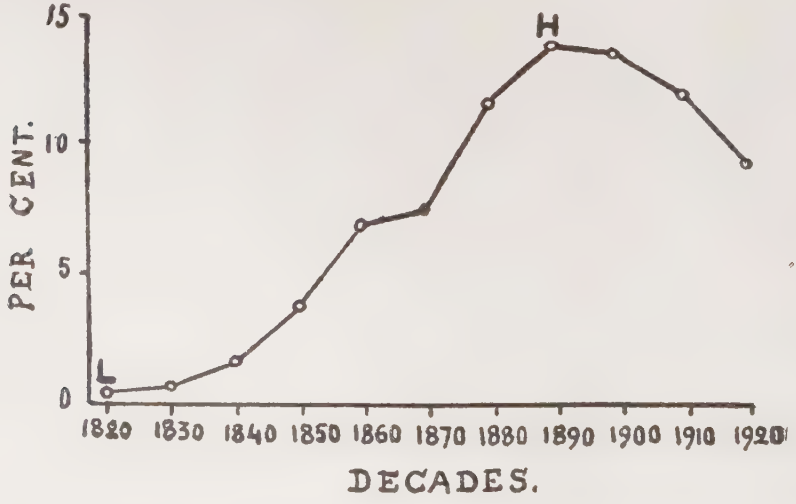

FIG. 32—PERCENTAGE OF EXERCISES IN FACTORING, FACTORS, AND MULTIPLES

TABLE 64

DISTRIBUTION OF EXERCISES IN FACTORING, FACTORS, AND MULTIPLES ACCORDING TO THE FIFTH IN WHICH THEY APPEARED IN THEIR RESPECTIVE BOOKS

DECADE	PER CENT OF EXERCISES IN FACTORING, FACTORS, AND MULTIPLES IN THE				
	FIRST FIFTH	SECOND FIFTH	THIRD FIFTH	FOURTH FIFTH	FIFTH FIFTH
1820	100.00**	.00*	.00*	.00*	.00*
1830	63.51**	29.73	6.76	.00*	.00*
1840	34.39	49.83**	9.47	6.32	.00*
1850	41.73	50.61**	.61*	6.08	.97
1860	22.74	71.55**	4.15	.42*	1.14
1870	23.68	59.64**	7.28	2.60*	6.81
1880	13.11	75.49**	7.19	1.15*	3.07
1890	3.66	62.73**	26.07	5.57	1.98
1900	7.03	64.08**	20.18	6.17	2.53*
1910	2.80	30.16	51.24**	13.09	2.72*
1920	2.27*	32.44	46.14**	11.74	7.42

*Minimum percentage for the decade.
**Maximum percentage for the decade.

Read table as follows: In the decade beginning in 1830, 63.51 per cent of the exercises in factoring, factors, and multiples appeared in the first fifth of their respective books; 29.73 per cent appeared in the second fifth; and 6.76 per cent, in the third fifth. None appeared in the last two fifths.

The percentages given in this table have been computed according to the method described on page ix–x.

TABLE 65

PERCENTAGE OF SIMPLE AND COMPLICATED EXERCISES IN FACTORING, FACTORS, AND MULTIPLES FOR EACH DECADE[1]

I FACTORING			II HIGHEST COMMON FACTOR AND LOWEST COMMON MULTIPLE		
DECADE	PER CENT OF EXERCISES		DECADE	PER CENT OF EXERCISES	
	SIMPLE	COMPLICATED		SIMPLE	COMPLICATED
1820	100.00**	.00*	1820	00.00	00.00
1830	94.44	5.56	1830	4.55*	95.45**
1840	87.86	12.14	1840	39.76	60.24
1850	82.33	17.67	1850	64.41	35.59
1860	79.05	20.95	1860	77.15	22.85
1870	86.91	13.09	1870	75.00	25.00
1880	57.56	42.44	1880	62.46	37.54
1890	50.81*	49.19**	1890	65.67	34.33
1900	52.94	47.06	1900	79.09	20.91
1910	64.88	35.12	1910	98.39	1.61
1920	83.37	16.63	1920	99.34**	.66*

*Minimum percentage for the entire period.
**Maximum percentage for the entire period.

Read table as follows:

I. In 1820–29, all the exercises in factoring were simple exercises. In 1830–39, 94.44 per cent of the exercises were simple and 5.56 per cent were complicated. See Fig. 33.

II. In 1820–29, there were no exercises in highest common factor and lowest common multiple. In 1830–39, 4.55 per cent of the exercises in these were simple and 95.45 per cent were complicated. See Fig. 34.

The types of exercises composing the two groups—simple and complicated—are listed on pages 116–117.

[1] Table 65 is given in two sections as the exercises in (1) factoring and (2) highest common factor and lowest common multiple are treated separately.

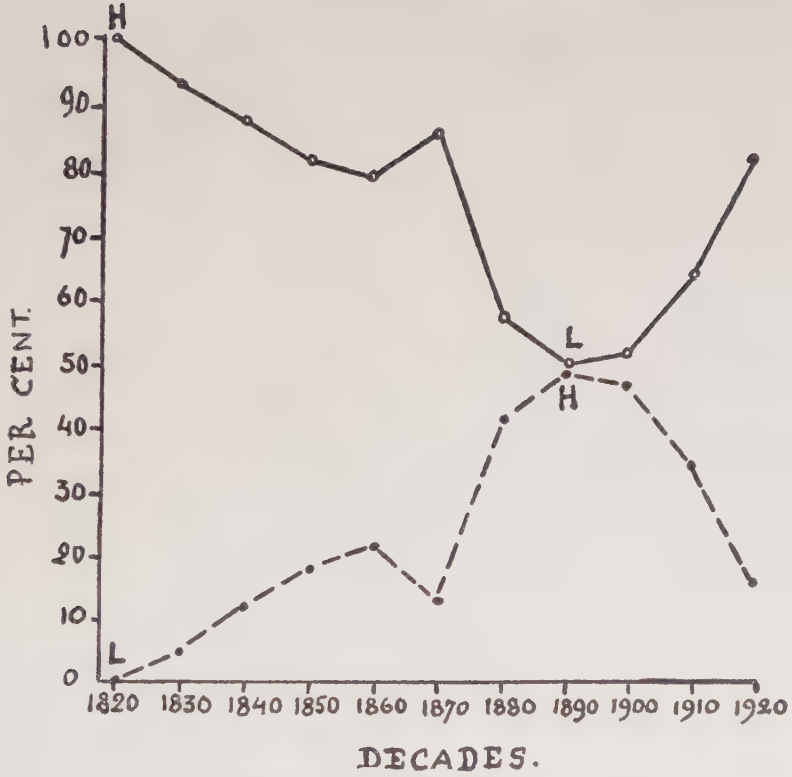

FIG. 33—COMPARISON OF SIMPLE AND COMPLICATED EXERCISES IN FACTORING

_____ Represents simple exercises in factoring.
— — — Represents complicated exercises in factoring.

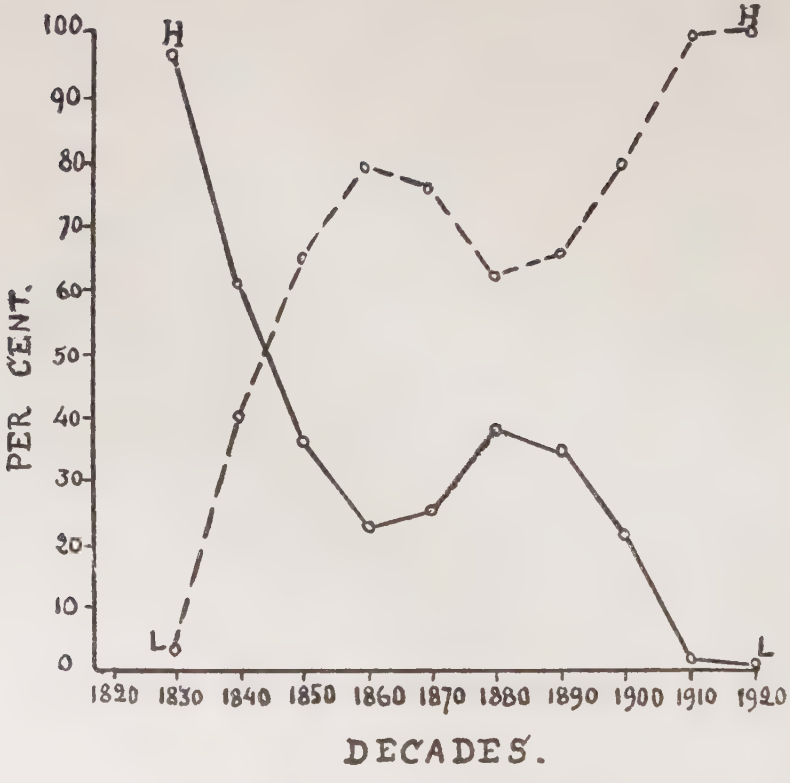

FIG. 34—COMPARISON OF SIMPLE AND COMPLICATED EXERCISES IN HIGHEST COMMON FACTOR AND LOWEST COMMON MULTIPLE

– – – – Represents simple exercises in highest common factor and lowest common multiple.

——— Represents complicated exercises in highest common factor and lowest common multiple.

TABLE 66

PERCENTAGE OF EXERCISES IN FACTORING, FACTORS, AND MULTIPLES DEVOTED TO FACTORING, HIGHEST COMMON FACTOR, AND LOWEST COMMON MULTIPLE, RESPECTIVELY

DECADE	PER CENT OF EXERCISES IN FACTORING, FACTORS, AND MULTIPLES DEVOTED TO		
	FACTORING	HIGHEST COMMON FACTOR	LOWEST COMMON MULTIPLE
1820	100.00**	00.00*	00.00*
1830	62.07	37.93**	00.00*
1840	62.78	21.08	16.14**
1850	75.03	12.27	12.69
1860	60.52*	24.84	14.64
1870	72.17	13.28	14.55
1880	71.91	16.01	12.07
1890	76.04	12.80	11.16
1900	79.63	10.81	9.56
1910	85.78	6.31	7.90
1920	91.85	2.01	6.14

*Minimum percentage for the entire period.
**Maximum percentage for the entire period.

Read table as follows: In 1820–29, there were no exercises in either highest ommon factor or lowest common multiple. In 1830–39, 62.07 per cent of che exercises in factoring, factors, and multiples were in factoring, and 37.93 were in highest common factor; there was no drill in lowest common multiple. See Fig. 35.

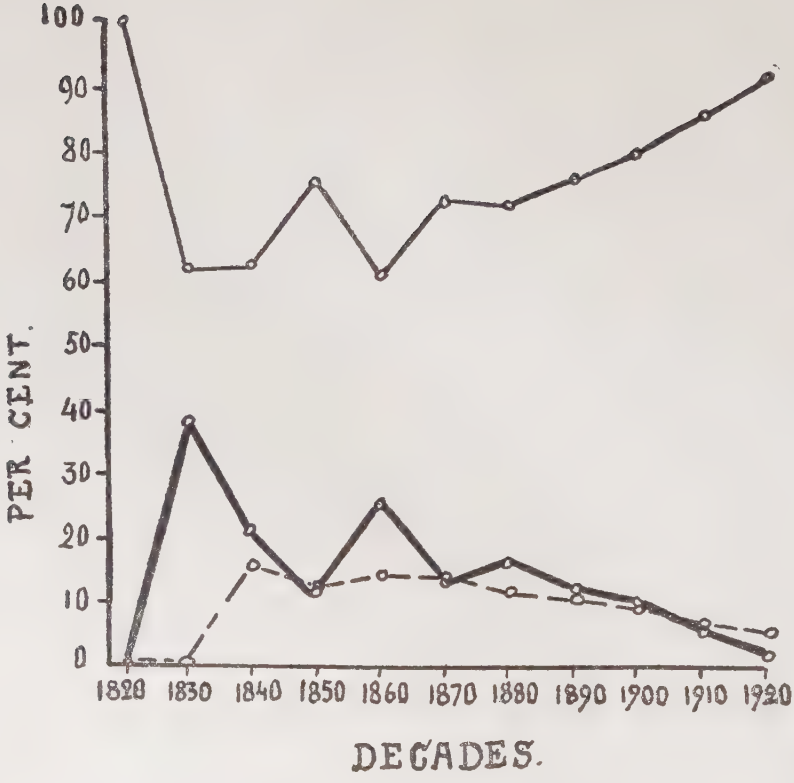

FIG. 35—COMPARISON OF FACTORING, HIGHEST COMMON FACTOR, AND LOWEST COMMON MULTIPLE

_____ Represents exercises in factoring.
_____ Represents exercises in highest common factor.
_ _ _ Represents exercises in lowest common multiple.

Discussion

Table 62 shows that the topic of factoring, factors, and multiples increased from decade to decade in mean number of exercises from .8 exercises in 1820–29 to 373.1 in 1910–19. The former is the minimum for the period and the latter the maximum.

In the first four decades, the mean number increased from .8 to 34.3 exercises. In the next decade (1860–69), the number increased further, amounting to 107.0 exercises. It increased over 100 exercises in both the eighties and the nineties, the number being 252.5 in the former decade and 359.8 in the latter. The first decades of the present century, however, increased only slightly—the first to 368.7 exercises, the second to 373.1. The number in the present decade amounts to 325.4 exercises, a decrease of 47.7 exercises or a loss of 12.8 per cent from 1910–19.

While the mean number of exercises in factoring, factors, and multiples continued to increase until the present decade was reached, Table 63 shows that the percentage devoted to them, steadily increasing in the nineteenth century, began to decrease in the first decade of the present century. In the first decade of the period examined, the exercises in this topic amounted to .21 per cent; in the nineties they totalled 13.74 per cent. The greatest increase took place in the eighties. In that decade the percentage changed to 11.56 from 7.58 in the decade preceding. In 1900–09, the percentage was only slightly lower than in 1890–99 and amounted to 13.46. In the next decade, however, it decreased to 11.91; and in the present decade amounts to 9.12. This is less than any percentage since the eighties.

As factoring, factors, and multiples increased in mean number and percentage, the largest percentage of drill gradually changed from the first to the second fifth. This is shown in Table 64. However, the tendency to postpone much of the drill to later in the course has not only continued to the present but has been intensified, with the result that the largest percentage is found in the third fifth in the present decade. This tendency may be clearly seen by examining the percentages in the various fifths. The first fifth decreased from 100.00 per cent in the first decade to 2.27 per cent in the present decade. The second fifth generally increased in the first seven decades, then decreased in the last

four. In 1830–39, 29.73 per cent of the exercises were in the second fifth; in 1880–89, 75.49; in the present decade, 32.44 per cent. The third fifth has varied, but the tendency on the whole has been to increase the percentage in it. In the last two decades the percentages in this fifth have been 51.24 and 46.14 per cent respectively. The third and fourth fifths have likewise tended to increase. Previous to the present decade, the minimum percentage was either in the fourth or in the last fifth in the majority of the cases. Now, on the other hand, it is in the first fifth.

In comparing the complexity and simplicity of exercises in factoring, factors, and multiples, the exercises in factoring have been separated from those in highest common factor and lowest common multiple. Table 65, Part I, shows that all the exercises in factoring were simple exercises in the first decade. In the second decade, 5.56 per cent were complicated. From then to the end of the century, the percentage in the latter increased, with the exception of the decade beginning in 1870. In the nineties the maximum was reached, 49.19 per cent being complicated and 50.81 simple. That decade together with the decade immediately preceding it and the one immediately following it form the period when complicated exercises in factoring were emphasized particularly.

In the decade, 1910–19, the complicated exercises in this topic dropped to 35.12 per cent. Simple exercises correspondingly increased to 64.88 per cent. The same tendency has continued in the present decade. At present complicated exercises amount to 16.63 per cent, less than half the percentage for 1910–19 and practically two-thirds less than the percentage in 1890–99. The percentage of both simple and complicated exercises in factoring in the present decade differs from the corresponding percentage in 1850–59 by only 1.04 points. On the whole, the percentages in this decade are more similar to those in the early decades of the past century than to those either in the last part of the nineteenth century or in the beginning of the present century.

As factoring increased in importance, it appears natural that finding the highest common factor and lowest common multiple by this process should increase and that the generally more

difficult methods of division should decrease. Table 65, Part II, shows that this is what happened in the first fifty years of the period. In 1830–39, when finding the highest common factor and lowest common multiple first appeared in the textbooks, 95.45 per cent of the exercises were to be solved by division and only 4.55 per cent by factoring. The percentage to be solved by division decreased greatly and the percentage to be solved by factoring correspondingly increased until after the decade of the sixties. In that decade, exercises in finding the highest common factor and lowest common multiple by division amounted to 22.85 per cent and exercises in finding them by factoring totalled 77.15 per cent, a decrease and increase respectively of 72.60 points from the percentages for 1830–39. Figure 34 shows, however, that the opposite tendency prevailed in the next two decades, in spite of the still increasing importance attached to factoring. The former tendency to lay more and more emphasis on solving by factoring reappeared in the nineties. Since then it has continued. In the present decade, 99.34 per cent of the exercises are to be solved by the generally simpler process of factoring, and only .66 per cent by division. These percentages indicate a complete reversal of the conditions existing in the decade of the thirties.

Drill in highest common factor did not appear until the second decade, according to the data in Table 66. In that decade, it amounted to 37.93 per cent of the exercises in factoring, factors, and multiples. All exercises in these three topics combined amounted only to 5.3 exercises in mean number. In the following decades of the century the percentages varied between 12.27 and 24.84 per cent. Since the nineties, however, highest common factor has decreased in percentage. In the present decade only 2.01 per cent of the exercises are in it.

Drill in lowest common multiple first appeared in 1840–49, as is shown in Figure 35. It then amounted to 16.14 per cent of the total drill in factoring, factors, and multiples. The percentages varied in the next two decades, but since the seventies have consistently decreased. It now amounts to 6.14 per cent. In only four decades has the percentage of exercises in lowest common multiples exceeded that in highest common factor.

Two of these decades were in the last century and two in the present century. They are respectively: 1850–59, 1870–79, 1910–19, and 1920–28.

Exclusive of the first decade, highest common factor and lowest common multiple combined constituted from 23.96 to 39.48 per cent of the drill in factors and multiples in the last century. Factoring, therefore, ranged from 60.52 to 76.04 per cent. In the present century, the percentage in factoring has increased from 79.63 per cent in the first decade, 1900–1909, to 91.85 per cent in the third decade, 1920–28. The percentage in highest common factor and lowest common multiple correspondingly decreased from 20.37 to 8.15 per cent.

Summary

Factoring, factors, and multiples combined reached their greatest importance, as judged by the emphasis placed on them, in the last decades of the nineteenth century and in the first decade of the twentieth century. In comparison with these decades, the present decade shows a decrease in mean number, percentage, and complexity of exercises. Drill also comes later in the course in the present decade.

XIII

Graphs

The development of graphs in the elementary course in algebra is the topic of this chapter. The discussion is preceded by Tables 67 to 72 which give the data on which the discussion is based and which deal with various aspects of the development of graphs.

Tables 67, 68, and 69 show respectively the mean number of exercises in graphs, the percentage of exercises devoted to them, and the distribution of these exercises. The first two tables also show the relative changes from decade to decade.

TABLE 67

Mean Number of Exercises in Graphs Per Book, Together With a Comparison of the Increase or Decrease in Each Decade

Decade	Mean number of exercises	Change from preceding decade		
		Number of exercises	Per cent increase or decrease	Rank in percentage of increase
1820–70	.0			
1880	6.8	+ 6.8		
1890	2.1*	− 4.7	− 69.1	4*
1900	51.0	+48.9	+1428.9	1**
1910	94.6	+43.6	+ 85.5	2
1920	128.3**	+33.7	+ 35.7	3

*Minimum or lowest since the appearance of graphs in 1880.
**Maximum or highest since the appearance of graphs in 1880.
Read table as follows: In the decade from 1910 to 1919, the mean number of exercises in graphs amounted to 94.6 exercises per book, an increase of 43.6 exercises or 85.5 per cent over the decade from 1900 to 1909. This decade (1910–1919) ranked second in the percentage of increase in the mean number of exercises in graphs.

Table 70 compares the percentage of simple and complicated exercises in graphs. For this purpose exercises in statistical graphs, graphs of equations, and graphs of positive and negative numbers have been considered simple exercises. Graphs of equations of the second or higher degree and graphs of complex and radical numbers compose the complicated group.

Tables 71 and 72 show the development of different phases of graph-work. In Table 71, exercises in constructing graphs are

compared with those in interpreting graphs. In Table 72 graphs of statistics are compared with graphs of equations, the percentage of exercises in each being given.

TABLE 68

PERCENTAGE OF EXERCISES IN GRAPHS, TOGETHER WITH A COMPARISON OF THE INCREASE OR DECREASE IN EACH DECADE

DECADE	EXERCISES IN GRAPHS			
	PER CENT	CHANGE FROM PRECEDING DECADE	CHANGE FROM 1880[1]	RANK IN PERCENTAGE OF EXERCISES
1820–70	.00			
1880	.31	+ .31		4
1890	.08*	− .23	− .23	5*
1900	1.86	+1.78	+1.55	3
1910	3.02	+1.16	+2.71	2
1920	3.60**	+ .58	+3.29	1**

*Minimum or lowest since the appearance of graphs in 1880.
**Maximum or highest since the appearance of graphs in 1880.

Read table as follows: In the decade from 1900 to 1909, exercises in graphs amounted to 1.86 per cent of the total exercises for the decade. This was an increase of 1.78 points over the percentage for the nineties and of 1.55 points over that for the eighties. This decade (1900–1909) ranked third in the emphasis placed on graphs. See Fig. 36.

[1] This is the first decade in which graphs appeared.

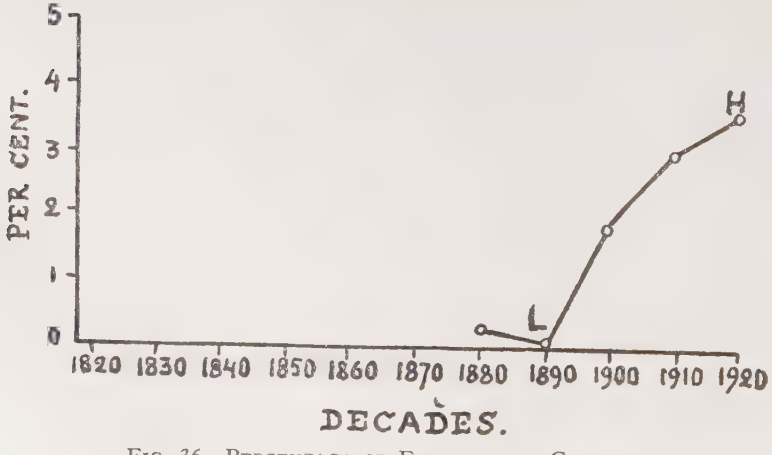

FIG. 36—PERCENTAGE OF EXERCISES IN GRAPHS

TABLE 69

DISTRIBUTION OF EXERCISES IN GRAPHS ACCORDING TO THE FIFTHS IN WHICH THEY APPEARED IN THEIR RESPECTIVE BOOKS

DECADE	PER CENT OF EXERCISES IN GRAPHS IN THE				
	FIRST FIFTH	SECOND FIFTH	THIRD FIFTH	FOURTH FIFTH	FIFTH FIFTH
1820–70	.00	.00	.00	.00	.00
1880	.00*	11.03	.00*	43.38	45.59**
1890	33.33	52.17**	14.49	.00*	.00*
1900	8.79	6.87*	28.46	18.11	37.78**
1910	10.43	9.92*	16.06	44.60**	18.99
1920	13.00	8.06*	17.61	30.21	31.08**

*Minimum percentage since the appearance of graphs in 1880.
**Maximum percentage since the appearance of graphs in 1880.

Read table as follows: In the decade from 1900 to 1909, 8.79 per cent of the exercises in graphs appeared in the first fifth of their respective books; 6.87 per cent, in the second fifth; 28.46 per cent, in the third fifth; 18.11 per cent, in the fourth fifth; and 37.78 per cent, in the last fifth.

The percentages in this table have been computed according to the method given on page ix–x.

TABLE 70

PERCENTAGE OF SIMPLE AND COMPLICATED EXERCISES IN GRAPHS FOR EACH DECADE

DECADE	PER CENT OF EXERCISES	
	SIMPLE	COMPLICATED
1820–70	00.00	00.00
1880	54.41*	45.59**
1890	100.00**	00.00*
1900	63.29	36.71
1910	84.89	15.11
1920	81.92	18.08

*Minimum percentage since the appearance of graphs.
**Maximum percentage since the appearance of graphs.

Read table as follows: There were no graphs until the decade of the eighties. In that decade 54.41 per cent of the exercises in graphs were simple exercises and 45.59 per cent were complicated. See Fig. 37.

The types of exercises that compose the two groups—simple and complicated—are listed on page 129.

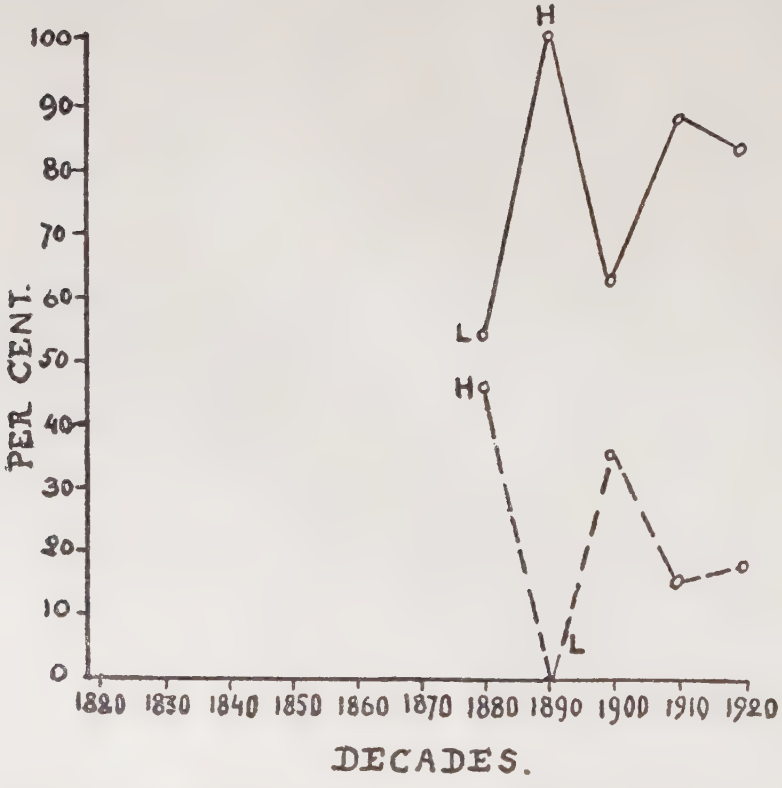

FIG. 37—COMPARISON OF SIMPLE AND COMPLICATED EXERCISES IN GRAPHS

_____ Represents simple exercises in graphs.
_ _ _ _ Represents complicated exercises in graphs.

TABLE 71

PERCENTAGE OF EXERCISES IN GRAPHS DEVOTED RESPECTIVELY TO THE CONSTRUCTION AND THE INTERPRETATION OF GRAPHS

DECADE	PER CENT OF EXERCISES IN GRAPHS DEVOTED TO	
	CONSTRUCTION	INTERPRETATION
1820–70	00.00	00.00
1880	87.50	12.50
1890	100.00**	00.00*
1900	82.83	17.17
1910	80.88	19.12
1920	69.21*	30.79**

*Minimum percentage since the appearance of graphs in 1880.
**Maximum percentage since the appearance of graphs in 1880.
Read table as follows: In 1910–19, 80.88 per cent of the exercises in the constructing and interpreting of graphs were in construction and 19.12 per cent were in interpretation. See Fig. 38.

TABLE 72

COMPARISON OF EXERCISES IN GRAPHS DEVOTED TO STATISTICAL GRAPHS AND GRAPHS OF EQUATIONS, RESPECTIVELY

DECADE	PER CENT OF EXERCISES DEVOTED TO	
	STATISTICAL GRAPHS	GRAPHS OF EQUATIONS
1820–70	0.00	0.00
1880	0.00*	100.00**
1890	0.00*	100.00**
1900	5.07	94.93
1910	15.53	84.47
1920	29.15**	70.85*

*Minimum percentage since the appearance of graphs in 1880.
**Maximum percentage since the appearance of graphs in 1880.
Read table as follows: In 1900–09 5.07 per cent of the exercises in statistical and equational graphs dealt with statistical graphs and 94.93 per cent with graphs of equations. See Fig. 39.

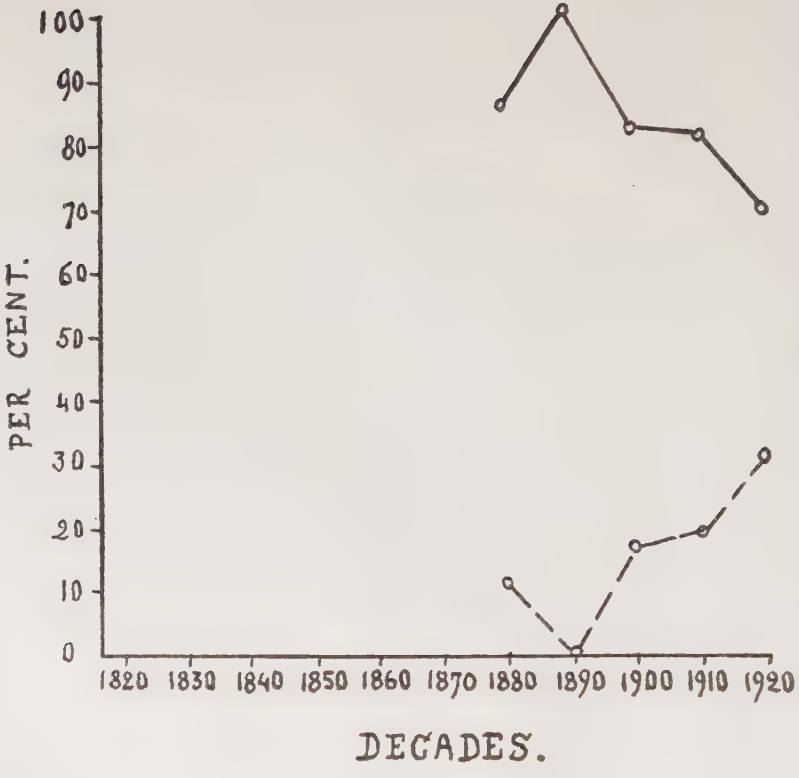

FIG. 38—COMPARISON OF EXERCISES IN CONSTRUCTION AND IN INTERPRETATION OF GRAPHS

_____ Represents exercises in the construction of graphs.
_ _ _ Represents exercises in the interpretation of graphs.

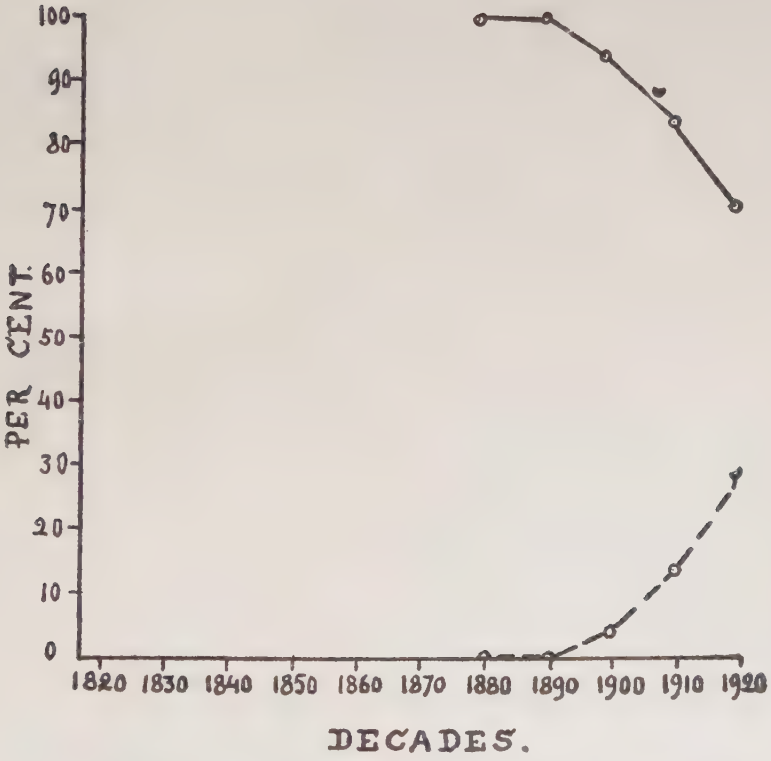

FIG. 39—COMPARISON OF STATISTICAL GRAPHS AND GRAPHS OF EQUATIONS

_ _ _ _ Represents exercises in statistical graphs.
_____ Represents exercises in graphs of equations.

Discussion

Judged by the textbooks examined, graphs did not appear in the course in elementary algebra previous to the last twenty years of the nineteenth century. Until the present century was reached, they apparently remained negligible in amount and percentage.

As Table 67 shows, the mean number of exercises amounted to 6.8 in 1880–89 and to 2.1 in 1890–99. This increased to 51.0, 94.6, and 128.3 exercises respectively in 1900–09, 1910–19, and 1920–28. Thus, work in graphical representation more than doubled in mean number of exercises since the beginning of the twentieth century.

As Table 68 and Figure 36 show, the percentage of exercises devoted to graphs amounted to .31 and .08 respectively in the last two decades of the nineteenth century. In the decade from 1900 to 1909, however, the exercises in graphing increased to 1.86 per cent. In the following decade they increased to 3.02 per cent. In the present decade they amount to 3.60 per cent. Thus the percentage of exercises devoted to graphs has almost doubled in the last three decades.

As the percentage of exercises in graphs amounted to less than half of one per cent preceding this century, the remainder of the discussion will stress primarily the development of graphs in the present century.

It is seen from Table 69 that since the beginning of this century, the largest percentage of exercises in graphs has been in either the fourth or the last fifth. The first fifth has, however, increased in percentage of exercises. In 1900–09, 8.79 per cent of the exercises in graphing were in this fifth. In the present decade 13.00 per cent comes in this fifth. In the second fifth, the percentage has ranged between 6.87 and 9.92.

Table 70 and Figure 37 show that in this century the largest percentage of complicated exercises in graphical representation occurred in the first decade. In the ten years from 1900 to 1909, 63.29 per cent of the exercises in graph-work were simple exercises and 36.71 were complicated. In the present decade the percentage of complicated drill in graphs amounts to 18.08.

This is a slightly larger percentage than in 1910–19, but approximately half the percentage for 1900–09.

Exercises in the interpretation of graphs have increased in each of the three decades of this century, according to Table 71 and Figure 38. Exercises in construction have correspondingly decreased in percentage. In 1900–09, 17.17 per cent of the exercises in graphs were in interpretation and 82.83 in construction. In the decade from 1910 to 1919, the percentage amounted to 19.12 in the former and 80.88 in the latter. In the present decade 30.79 per cent are in interpretation and 69.21 in construction.

In Table 72 and Figure 39 are compared the exercises in statistical graphs and graphs of equations. It is seen that statistical graphs did not appear in the elementary course until this century. In 1900–09 they amounted to 5.07 per cent of the total number of statistical and equational graphs. In the following decade the percentage more than tripled, amounting to 15.53 per cent. Graphs of equations correspondingly decreased from 94.93 to 84.47 per cent. In the present decade the percentages are 29.15 for graphs of statistics and 70.85 for graphs of equations. Thus, at present, the latter are stressed to a greater extent than previously, the percentage in the present decade being between five and six times as large as in the first decade of the century.

Summary

Graphs were omitted from the course until the eighties and were practically negligible until the beginning of the twentieth century. Since the 1900 decade, however, they have nearly doubled in percentage and more than doubled in mean number. In the present decade, the number of exercises total 128.3 on the average and comprise 3.60 per cent of all the exercises for the decade. At the present time work in graphs is found in all the fifths, the majority of the exercises being in the last two fifths of their respective books. Complicated exercises are stressed to a smaller extent now than they were at the beginning of the century, less than one-fifth of the exercises being complicated in the present decade.

Slightly over two-thirds of the exercises in graphs are in their construction. This is less than previously. Correspondingly the percentage in interpretation has increased. Statistical graphs have also increased greatly. There is slightly more than one statistical graph for every three graphs of equations in the present decade. On the other hand, in the first decade of this century the ratio was approximately one to twenty.

XIV

Summary and Comparisons

Chapter 14 is divided into three parts. In Part A is given a résumé of the facts previously brought out concerning the topics of elementary algebra, as revealed by the textbooks examined. This is followed by Part B in which the central tendencies are summarized and present changes in content are viewed in the light of the past. Part C contains a comparison of the findings with the recommendations made by the National Committee on Mathematical Requirements in its report, entitled the *Reorganization of Mathematics in Secondary Education*, first published in 1923.

Part A

Part A of this summary is devoted to a résumé of the facts brought out in Chapters 2 to 13 inclusive. First, in Tables 73 and 74, the general topics of elementary algebra are presented from the points of view of their relative importance as judged by the percentage of exercises devoted to them and of their ranks based on these percentages. The topics are next briefly discussed from the viewpoint of the distribution of their drill in the books, Table 75 giving a summary of their distribution from the decade beginning in 1820 to the present. The periods of greatest complexity and simplicity are summarized in Table 76. This shows for each topic the four decades having the largest percentages of complicated and simple exercises.

The mean number of exercises in each topic is given in Table 77. Following this is a discussion of the periods of largest and smallest increases in the topics as a whole. The general tendencies in the percentages for each topic are noted in connection with Table 78, which shows the percentages for each decade. These tendencies are divided into those for the nineteenth century and those for the twentieth century.

TABLE 73

TOPICS OF ELEMENTARY ALGEBRA, 1820–1928, ARRANGED TO SHOW THEIR RELATIVE IMPORTANCE AS JUDGED BY THE PERCENTAGE OF EXERCISES DEVOTED TO EACH

TOPICS	DECADES										
	1820	1830	1840	1850	1860	1870	1880	1890	1900	1910	1920
Equations	A	A	A	A	A	A	A	A	A	A	A
Roots and Radicals	B	C	B	C	C	C	C	B	B	D	D
Fractions	C	C	C	C	D	C	D	D	D	D	D
General Equations and Formulas	C	D	D	C	D	D	D	D	D	D	C
Multiplication	D	C	C	C	C	C	C	C	C	C	C
Addition and Subtraction	D	C	D	D	D	D	D	E	D	D	D
Division	D	D	D	D	E	D	D	D	E	D	D
Proportions and Progressions	E	D	D	D	D	D	D	D	E	E	E
Substitution	E	+	E	E	E	E	E	E	E	E	E
Negative Exponents	E	+	+	+	E	E	E	E	E	+	+
Parentheses	+	+	+	+	E	E	E	E	E	E	E
Factoring, Factors and Multiples	+	+	E	E	D	D	C	C	C	C	D
Graphs	−	−	−	−	−	−	+	+	E	E	E

LEGEND: A—Percentage of exercises amounts to 20 per cent or more of total.
B—Percentage of exercises lies between 15 and 19.99 per cent of total.
C—Percentage of exercises lies between 10 and 14.99 per cent of total.
D—Percentage of exercises lies between 5 and 9.99 per cent of total.
E—Percentage of exercises lies between 1 and 4.99 per cent of total.
+—Percentage of exercises amounts to less than one per cent of total.
−—Entire omission of exercises.

Read table as follows: Exercises in factoring, factors, and multiples amounted to less than one per cent of the total number of exercises in each of the first two decades. The percentage of exercises ranged between 1 and 4.99 per cent in the following two decades; between 5 and 9.99 per cent in the decades of the sixties and seventies; between 10 and 14.99 per cent in the decades from 1880 to 1919; and between 5 and 9.99 per cent in the last decade (1920–28).

Discussion

It is shown in Table 73 that the general topics[1] of elementary algebra, with the exception of graphs, have been in the course since the beginning of algebra as a high school subject. This table shows, however, that drill in parentheses and the combined topics of factoring, factors, and multiples was so slight (less than one per cent of the total) in the earlier decades as to be almost negligible. This is also true of negative exponents for a period of thirty years beginning approximately in the decade of the thirties and for the period since the second decade of the present century. Graphs, which did not appear preceding the eighties, remained negligible until the present century, since the beginning of which they have received between 1 and 4.99 per cent of the drill.

Equations have received most of the drill—over 20 per cent in every decade. The topic of roots and radicals received from 10 to 19.99 per cent until the last two decades in which interval it has received less than this amount. Multiplication has usually received between 10 and 14.99 per cent of the drill, as did fractions in the first half of the nineteenth century. Factoring, factors, and multiples, negligible from the standpoint of drill in the first twenty years of the period, received together between 10 and 14.99 per cent of the drill for a period of approximately 40 years beginning about 1880.

The topics of (1) general equations and formulas and (2) addition and subtraction, are the only other topics that have received over 10 per cent of the drill at any time during the period included in the study—the former in three widely separated decades, 1820–29, 1850–59, and 1920–28, the latter in one decade, 1830–39. The remaining topics have been given relatively less emphasis, judging from the number of exercises devoted to them. Division has generally received between 5 and 9.99 per cent of the drill. Proportions and progressions received between 5 and 9.99 per cent in the decades from 1830 to 1899. In the twenties, this topic received less drill. In the present century also the percentage has been lower. Parentheses, graphs, negative exponents, and substitution have had the smallest percent-

[1] Changes in the development of specific phases of the general topics are not discussed here. See the chapter for each topic for these.

TABLE 74

TOPICS OF ELEMENTARY ALGEBRA, 1820–1928, ARRANGED TO SHOW THEIR RELATIVE RANKS, BASED ON THE PERCENTAGE OF EXERCISES DEVOTED TO EACH

TOPICS		DECADES										
		1820	1830	1840	1850	1860	1870	1880	1890	1900	1910	1920
Equations	O	1	1	1	1	1	1	1	1	1	1	1
Roots and Radicals	−	2	3	2	2	2	2	2	2	2	4	4
Fractions	−	3	4	4	3	4	4	5	5	5	6	6
General Equations and Formulas	+	4	7	5	5	6	6	6	6	6	5	2
Multiplication	+	5	2	3	4	3	3	4	4	4	2	3
Addition and Subtraction	−	6	5	7	7	8	8	8	9	7	7	7
Division	−	7	6	6	8	9	9	9	7	8	8	8
Proportions and Progressions	−	8	8	8	6	5	5	7	8	9	9	9
Substitution	−	9	9	9	10	10	10	12	10	10	11	11
Negative Exponents	−	10	12	11	11	11	11	10	12	13	13	13
Parentheses	−	11	11	12	12	12	12	11	11	12	12	12
Factoring, Factors, and Multiples	+	12	10	10	9	7	7	3	3	3	3	5
Graphs	+	13*	13*	13*	13*	13*	13*	13	13	11	10	10

* Exercises entirely omitted.

Legend: 1, 2, . . . 13 indicate the ranks, based on the percentage of exercises, 1 being the highest rank and 13 the lowest. +, O, and − indicate respectively, a higher rank, the same rank, and a lower rank in the present decade as compared with the first decade.

Read table as follows: Factoring, factors, and multiples have a higher rank in the present decade than they did in the first decade (1820–29). They ranked twelfth in the twenties, tenth in the next two decades, ninth in the decade of the fifties, seventh in the sixties and seventies, third in the interval between 1880 and 1919, and fifth in the last decade.

ages of exercises. None of these topics reached five per cent in any decade and all have been negligible in the course at one time or another.

Discussion

In Table 74 the topics are arranged to show their ranks, based on the percentage of exercises in each. Equations, it is seen, have always been first in rank. Roots and radicals were usually second in rank previous to the second decade of the present century, the only exception being in the decade from 1830 to 1839. In the second decade of the present century, they dropped to fourth in rank and in the present decade are still fourth. Fractions have also decreased in rank. They were third in the decades of the twenties and the fifties, fourth in the other decades of the nineteenth century until the decade of the eighties, and fifth in the interval between 1880 and 1909. Since the decade beginning in 1910 they have been sixth.

Multiplication and addition and subtraction combined are both higher in rank now than they were in the latter decades of the nineteenth century. Multiplication was fourth in the decades beginning in 1880, 1890, and 1900. In the decade, 1910–19, it increased to second, and in the present decade is third—the same rank as in the decades, 1840–49 and 1860–79. The topic of addition and subtraction, which ranked eighth in the decades between 1860 and 1889 and ninth in the decade of the nineties, has been consistently seventh since the beginning of the present century. Thus it is higher in rank than it was in the interval between 1860 and the end of the century, but it is still lower than in the decades of the twenties and thirties, when it ranked respectively sixth and fifth. Division has been eighth since the beginning of the twentieth century. Previously it varied between sixth and ninth, having its lowest rank—ninth—in the decades between 1860 and 1889.

One of the largest changes in rank has been that in proportions and progressions. This topic increased from eighth in the first decade to fifth in the sixties. It remained fifth in the following decade and then gradually decreased, holding the same rank at the end of the century as in the beginning of the period. Since the decade beginning in 1900 it has been ninth.

TABLE 75

TOPICS OF ELEMENTARY ALGEBRA, 1820–1928, ARRANGED TO SHOW THE FIFTHS OF THE BOOKS IN WHICH WERE FOUND THE LARGEST PERCENTAGES OF EXERCISES OF EACH TOPIC.

TOPICS	DECADES										
	1820	1830	1840	1850	1860	1870	1880	1890	1900	1910	1920
Addition and Subtraction	1	1	1	1	1	1	1	1	1	1	1
Parentheses	1	1	1	1	1	1	1	1	1	2	2
Substitution	1	1	3	1	1	1	1	1	1	1	1
Multiplication	1	1	1	1	1	1	1	1	1	2	2
Division	1	1	1	1	2	1	1	2	2	2	2
Factoring, Factors, and Multiples	1	1	2	2	2	2	2	2	2	3	3
Fractions	2	2	2	2	2	2	2	3	2	3	3
Negative Exponents	3	3	3–	2	2–	2–	4	4	4	5	5
Roots and Radicals	3	3–5–	3–	4	4	4	4	4	4	5	5
General Equations and Formulas	4–	5	5	5	5	5	5	5	5–	5–	5–
Proportions and Progressions	4	5	5	5	5	5	5	5	5	5	5
Equations	5–	4–	4–	3–	5–	3–	3–	5–	3–	5–	5–
Graphs	0	0	0	0	0	0	5	2	5–	4	5–

Legend: 1, 2, 3, 4, and 5 indicate the fifths in which were found the largest percentages of exercises for each topic, 1 indicating the first fifth and 5 the last fifth. A minus sign after the number indicates that the percentage in that fifth was less than 40 per cent.

Read table as follows: In factoring, factors,and multiples, the largest percentage of exercises for the decades, 1820–29 and 1830–39, appeared in the first fifth of their respective books. In the decades between 1840 and 1909 the largest percentage came in the second fifth. In each of the last two decades, the largest percentage has been in the third fifth.

The percentages that form the basis of this table have been computed according to the method described on page ix–x.

The largest change has been in factoring, factors, and multiples. This topic increased from twelfth or last in the first decade to third in the decade of the eighties. It remained third until the present decade, in which it dropped to fifth in rank.

Substitution is slightly lower in rank now than it generally was previous to the last two decades. It ranked ninth in the first three decades, and tenth in those between 1850 and 1909 except in the eighties when it was twelfth. It has ranked eleventh in the last two decades.

The topic lowest in rank has varied. Parentheses and negative exponents, however, have been the lowest in eight of the eleven decades, the former in the interval between 1840 and 1879, and the latter in the thirties and since the beginning of the present century. In the first decade, 1820–29, the lowest rank was held by factoring, factors, and multiples. Graphs, now tenth, ranked last in the first two decades of their appearance, *i.e.*, in the last twenty years of the nineteenth century.

Discussion

Table 75 gives a general view of the distribution of the drill in the various topics, as revealed by the textbooks examined. It does not, however, show the scattering of the drill nor necessarily mean that the majority of the exercises for the topic appeared in the fifth given. In nearly 85 per cent of the cases, the percentages given amount to 40 per cent or more of the total drill in the topic for the decade.

The table shows that addition and subtraction combined has not varied in the fifths in which the largest percentages of its exercises appeared. This is the only topic of which this is true. Since the beginning of the period, its largest percentage has been in the first fifth. In substitution the largest percentage occurred in the first fifth with the exception of the decade from 1840 to 1849.

The largest percentages of the exercises in parentheses and multiplication occurred in the first fifth until the last two decades. Since then they have been in the second fifth. In division, the largest percentage generally occurred in the first fifth until the last decade of the century was reached. Since and including

the decade of the nineties, it has been in the second fifth. Fractions also show a tendency towards later drill now. The largest percentage of fractions appeared in the second fifth until the nineties. In the last two decades it has been in the third fifth.

Formulas and general equations, and proportions and progressions have had the largest percentages of their exercises in the last fifth with the exception of the first decade. However, in the case of the former the largest percentage in the last three decades has been less than 40 per cent. Thus, although the largest number is still in the last fifth, the total there is less than previously. Graphs, which did not enter into the elementary course preceding the eighties, have varied between the second and the last fifths. Equations have varied between the third and the last fifths, but in no case has the largest percentage amounted to 40 per cent.

The three topics that have shown the largest consistent changes in distribution are (1) factoring, factors, and multiples, (2) roots and radicals, and (3) negative exponents. In the first of these the largest percentage has gradually changed from the first to the third fifth. In negative exponents, the largest percentage changed from the third to the second fifth in the middle of the century. Since then, however, this topic has changed in the other direction, the largest percentage now being in the last fifth. In the case of roots and radicals, the largest proportion of the exercises came in the third fifth in the twenties and forties. In the thirties the percentage was the same in the last as in the third fifth. In the decades between 1850 and 1909 the greatest number occurred in the fourth fifth. In the last two decades, it has been in the last fifth.

Discussion

Table 76 shows that the decade from 1880 to 1889 represents the beginning of the period lasting about thirty years, in which the percentage of complicated exercises was consistently high in the majority of topics. In fact, the decade of the eighties may be characterized as the decade of greatest complexity in the entire period. Only one topic did not have one of its four highest percentages of complicated exercises in that decade. Further-

TABLE 76

TOPICS OF ELEMENTARY ALGEBRA, 1820–1928, ARRANGED TO SHOW THE PERIODS OF GREATEST COMPLEXITY AND SIMPLICITY

TOPICS		DECADES										
		1820	1830	1840	1850	1860	1870	1880	1890	1900	1910	1920
Equations	+	..	s	..	..	c	s	C	c*	c	s	S
Division	+	..	s	..	..	c	s	C	c*	c	s	S
Fractions	−	s	S	s	..	..	s	c	C	c*	c	..
Factoring	−	S	s	s	..	..	s	c	C	c*	c	..
Roots and Radicals	+	C	s	..	s	..	..	c	c	c*	s	S
Multiplication	+	..	s	s	..	c	..	C	c	c	s*	S
Parentheses	−	S	S	S	..	..	..	c	c	C	c*	s
Graphs	+	O	O	O	O	O	O	C	S	c	s*	..
Addition and Subtraction	+	C	c	c	..	..	c	s	s	..	s*	S
Factors and Multiples	+	O	C	c	c	..	..	c	..*	s	s	S
Proportions and Progressions	+	s	s	C	c	c	..	c	..*	..	S	S

Legend: C—Indicates the decade in which the percentage of complicated exercises was highest for that topic.
c—Indicates the three decades, next highest in percentage of complicated exercises. (One decade[1] for graphs).
S—Indicates the decade in which the percentage of simple exercises was highest for that topic.
s—Indicates the three decades, next highest in percentage of simple exercises.[1]
*—Indicates the decade in which started the present movement towards simplicity.
+—Indicates that the percentage of simple exercises is greater in the present decade than it was in the first decade.
−—Indicates that the percentage of simple exercises is less in the present decade than it was in the first decade.

Read table as follows: The highest percentage of simple exercises in factoring appeared in the first decade. The three next highest percentages came in the decades of the thirties, forties, and seventies. The highest percentage of complicated exercises appeared in 1890, the three next highest percentages being in the decades beginning in 1880, 1900, and 1910. The present movement towards simplicity began in the decade from 1900 to 1909. The percentage of simple exercises in factoring is less in the present decade than it was in the first decade.

The types of exercises that compose the two groups—simple and complicated—are listed in the chapter for each topic.

[1] One decade for graphs as this topic did not appear until the eighties.

more, four of the seven topics, each having its highest percentage of such exercises in the interval between 1880 and 1909 had this percentage in the eighties.

The topics that were consistently high in relative number of complicated exercises in the period beginning in the eighties and ending in the first decade of the present century are: equations, roots and radicals, fractions, multiplication, division, parentheses, and factoring. The most outstanding exception is the topic of addition and subtraction. The period of greatest complexity in it came earlier in the nineteenth century. Proportions and progressions, and common factors and multiples also had relatively more complicated exercises previous to this time.

The present trend towards greater simplicity began, however, to manifest itself in this period of complexity. This is indicated by the asterisks in Table 76. It did not gain much momentum until later, but now the reaction against complexity is present in all topics. In fact, seven topics have their highest percentages of simple exercises in the present decade. This trend towards increased simplicity is merely the renewal of a tendency first evident in the decade from 1830 to 1839, when the percentage of simple exercises in eight of the ten topics of the course were relatively high. This earlier reaction was of short duration. It reappeared in diminished form in the decade beginning in 1870. The first period relatively high in the proportion of simple exercises occurred, therefore, in the thirties and the second in the seventies. The present and most pronounced period of such exercises comprises the present decade and the one immediately preceding it.

Discussion

Table 77 shows that the general tendency in the topics of elementary algebra has been to increase the mean number of exercises from decade to decade. The comparatively few decreases are scattered among many topics. In fact, only two topics (multiplication and fractions) have increased in mean number of exercises in every decade without exception. The mean number increased in every decade but one in seven topics, namely: equations; general equations and formulas; addition

TABLE 77

TOPICS OF ELEMENTARY ALGEBRA, 1820–1928. MEAN NUMBER OF EXERCISES IN EACH

TOPICS		DECADES										
		1820	1830	1840	1850	1860	1870	1880	1890	1900	1910	1920
Multiplication	A	37.1*	78.1	104.0	110.0	167.0	192.4	239.3	311.6	314.1	386.8	435.5**
Fractions	A	51.1*	72.4	91.6	115.3	143.3	185.1	213.5	216.2	232.4	257.4	314.8**
Equations	B	137.3*	211.1	288.6	320.7	512.0	*468.6*	602.2	691.2	758.7	935.2	1105.8**
General Equations and Formulas	B	40.9*	53.0	78.8	107.5	131.8	144.3	201.2	*187.3*	216.3	289.4	525.4**
Addition and Subtraction	B	34.6*	62.5	*56.5*	68.1	90.0	107.1	121.1	128.6	153.0	252.8	293.1**
Division	B	27.0*	53.2	59.0	61.7	70.3	94.2	120.5	137.3	*126.0*	158.6	165.6**
Parentheses	B	.9*	2.1	4.8	6.8	19.0	21.8	36.9	50.5	*44.8*	68.5	69.9**
Factoring, Factors, and Multiples	B	.8*	5.3	14.3	34.3	107.0	126.9	252.5	359.8	368.7	373.1**	*325.4*
Graphs	B	.0	.0	.0	.0	.0	.0	6.8	*2.1**	51.0	94.6	128.3**
Roots and Radicals	C	68.6*	74.6	150.4	*126.0*	194.8	220.0	313.0	424.6	426.3**	*294.6*	334.9
Negative Exponents	C	4.3	*1.6**	7.1	8.9	21.6	22.4	38.4	41.8	42.2**	*17.3*	21.9
Proportions and Progressions	D	12.9*	33.8	54.3	77.0	114.8	152.8	163.2**	*134.3*	*102.7*	*97.5*	149.3
Substitution	D	9.1	*5.6**	16.5	*10.7*	24.5	26.2	36.0	64.8	*54.3*	81.1	81.8**
Unclassified		.8	7.4	9.1	14.6	44.1	56.0	41.1	57.5	66.6	115.8	141.1

* Minimum for the period in that topic. ** Maximum for the period in that topic.

Legend: A—Mean number of exercises increased in every decade.
B—Mean number of exercises increased in every decade but one.
C—Mean number of exercises increased in every decade but two.
D—Mean number of exercises increased in every decade but three.

Italic Figs.—Mean number of exercises is less than in preceding decade.

Read table as follows: The mean number of exercises in multiplication increased in every decade. In the first decade, the mean number amounted to 37.1 exercises; in the thirties, to 78.1; in the forties, to 104.0, etc. Negative exponents increased in every decade but two. In the first decade, the mean number in this topic amounted to 4.3 exercises. In the thirties, it totalled 1.6 exercises, a decrease from the preceding decade, etc.

and subtraction; division; parentheses; factoring, factors and multiples; and graphs. The remaining topics decreased in two or three decades. These topics are roots and radicals, negative exponents, proportions and progressions, and substitution. Proportions and progressions decreased in the interval between 1890 and 1919, but made up the greater part of this decrease in the present decade. Roots and radicals decreased greatly in the decade from 1910 to 1919, and are still appreciably lower than in the first decade of this century. In the last two decades negative exponents amounted to 17.3 and 21.9 exercises respectively, approximately half the mean number for the decade from 1900 to 1909 and less than in any decade since the seventies.

The percentages of increase in the topics have generally been greatest in the decades from 1830 to 1839 and from 1860 to 1869. In the decade, 1900–09, the majority of the topics increased less than 5 per cent. In both the thirties and the sixties, however, four topics increased between 50 and 100 per cent and four others increased over 100 per cent. The largest increases in number of exercises took place in the decades of the sixties and the eighties, the increase in all topics combined amounting to 555.4 exercises in the sixties and 510.7 in the eighties. By the decade, 1860–69, every topic in the course of the time had more than twice the number of exercises appearing in the twenties. Two decades later every topic except two (substitution and addition and subtraction) had more than four times the number occurring in the first decade. At present the mean number of exercises in several topics is six or more times the number for the decade from 1820 to 1829. In no case is it less than four times as large.

General Tendencies Shown in Table 78

Equations

Nineteenth century: The percentage generally decreased.

Twentieth century: The percentage increased.

Roots and Radicals

Nineteenth century: The percentage varied in the first three decades, decreased in the fifties and sixties, then increased to the end of the century.

TABLE 78

TOPICS OF ELEMENTARY ALGEBRA, 1820–1928; PERCENTAGE OF EXERCISES IN EACH

TOPICS		DECADES										
		1820	1830	1840	1850	1860	1870	1880	1890	1900	1910	1920
Equations	L	35.71†	34.74	33.72	33.62	33.91	28.00	27.57	26.38*	27.68	29.85	30.99
Roots and Radicals	L	17.84†	12.28	17.57	13.21	12.98	13.15	14.33	16.20	15.55	9.40	9.39*
Fractions	L	13.29†	11.91	10.70	12.09	9.49	11.06	9.77	8.25	8.48	8.21*	8.82
Addition and Subtraction	L	9.00	10.28†	6.60	7.14	5.96	6.40	5.54	4.91*	5.58	8.07	8.22
Division	L	7.02	8.75†	6.89	6.46	4.66	5.63	5.51	5.24	4.60*	5.06	4.64
Substitution	L	2.37	.93*	1.93	1.12	1.63	1.56	1.65	2.47	1.98	2.59†	2.29
Negative Exponents	L	1.12	.27*	.83	.93	1.40	1.34	1.76†	1.59	1.54	.55	.61
Formulas and General Equations	H	10.64	8.72	9.20	10.56	8.74	8.62	9.21	7.15*	7.89	9.24	14.73†
Multiplication	H	9.65*	12.85†	12.15	11.53	11.06	11.50	10.96	11.89	11.46	12.35	12.21
Proportions and Progressions	H	3.36	5.56	6.34	8.07	8.88	9.13†	7.47	5.13	3.75	3.11*	4.18
Parentheses	H	.23*	.34	.55	.71	1.26	1.30	1.69	1.93	1.64	2.18†	1.96
Factoring, Factors, and Multiples	H	.21*	.87	1.66	3.59	7.09	7.58	11.56	13.74†	13.46	11.91	9.12
Graphs	H	.00	.00	.00	.00	.00	.00	.31	.08*	1.86	3.02	3.60†
Unclassified	H	.21	1.21	1.06	1.53	2.92	3.35	1.88	2.19	2.43	3.70	3.96

* Minimum percentage since the appearance of the topic in the course.

† Maximum percentage since the appearance of the topic in the course.

Legend: L—Percentage for 1920 is lower than that for 1820.

H—Percentage for 1920 is higher than that for 1820.

Read table as follows: The percentage of exercises in equations is less in the present decade than it was in the first decade. In the decade from 1820 to 1829, 35.71 per cent of the exercises in the textbooks examined were exercises in equations. In the thirties the percentage was 34.74, etc.

Twentieth century: The percentage decreased.

Fractions

Nineteenth century: The percentage decreased with the exception of the fifties and seventies.

Twentieth century: The percentage varied between 8 and 9 per cent.

Addition and Subtraction

Nineteenth century: The percentage generally varied but on decreasing levels, hence the tendency was to decrease the percentage.

Twentieth century: The percentage increased.

Division

Nineteenth century: The percentage decreased except in the thirties and seventies.

Twentieth century: The percentage varied between 4+ and 5+ per cent.

Substitution

Nineteenth and twentieth centuries: The percentage varied between .93 and 2.59 per cent.

Negative Exponents

Nineteenth century: The percentage decreased in the thirties, then generally increased until the last decade of the century was reached. There was a slight decrease in the nineties.

Twentieth century: The percentage generally continued to decrease.

General Equations and Formulas

Nineteenth century: The percentage decreased in the thirties, increased in the forties and fifties, then varied to the end of the century.

Twentieth century: The percentage increased.

Multiplication

Nineteenth century: The percentage increased in the thirties, this increase being more or less maintained to the end of the century.

Twentieth century: The increase of the thirties still maintained.

Proportions and Progressions

Nineteenth century: The percentage increased in the first six decades, but decreased in the eighties and nineties.

Twentieth century: The tendency to decrease continued in the first two decades, the percentage increasing in the third decade.

Parentheses

Nineteenth century: Slight but continual increases in percentage.

Twentieth century: Slight decreases in the first and third decades; a slight increase in the second decade.

Factoring, Factors, and Multiples

Nineteenth century: Percentage increased in every decade.

Twentieth century: Slight decrease in percentage in the first decade, more pronounced decreases in the last two decades.

Graphs

Nineteenth century: Graphs did not appear until the eighties. The percentage varied in the eighties and nineties, but remained less than half of one per cent.

Twentieth century: The percentage increased in every decade.

Part B—Present Changes in the Light of the Past

Judging from the tendencies of the past as revealed by this study, the present changes in the content of elementary algebra may be broadly classified as resulting from:

1. Tendencies beginning early in the history of algebra as a high school subject and continuing more or less to the present.

2. Tendencies also beginning early in the history of high school algebra, but appearing only intermittently in the last century. Their appearances formed, in reality, a series of reactions against prevailing conditions.

3. Tendencies first appearing thirty or forty years ago and opposed to tendencies prevalent before that time. Changes resulting from these frequently make present conditions similar in certain respects to those existing in the earlier decades of the nineteenth century.

4. Tendencies also recently appearing in elementary algebra but leading to changes that may be termed real innovations in elementary algebra.

The changes resulting from tendencies in the first three groups are much more numerous than those resulting from the last group. In fact, very few of the changes in the content of elementary algebra, as revealed by this study, may with propriety be called innovations. There are, however, three notable changes which have no precedents in the past. One is the introduction of graphs into the elementary course. Graphs first appeared in the eighties but were negligible in amount until the present century. Since the beginning of the twentieth century they have increased in number and percentage. Statistical graphs appeared later than graphs of equations, but now amount to approximately 30 per cent of the exercises in graphs. The second change is the total elimination of progressions from the course of the last two decades. The third is the introduction of trigonometric ratios into the course of the present decade.

The increase in the mean number of exercises is a manifestation of the tendency to increase that has existed in every decade since the beginning of the period studied. In the thirties and again in the sixties the mean number increased over 58 per cent. The increases in the other decades varied between 11.4 and 40.8 per cent with the exception of the decade from 1900 to 1909, which increased only 4.6 per cent over the decade preceding it. At present the total mean number amounts to 3567.8 exercises per book. This is an increase of 13.9 per cent over the decade beginning in 1910. As has been the case in all other decades the increase has been mostly in tool and verbal exercises.

The changes in the distribution of the exercises in many of the topics also followed trends generally first evident in the earlier decades of the last century. The gradual postponement of a

large amount of drill in roots and radicals, fractions, negative exponents, division, and factoring, factors, and multiples to later in the course are instances of this. This is not true, however, of the changes in the distribution of the drill in addition and subtraction, parentheses, or multiplication. These changes apparently did not begin until more recent times.

Among the tendencies that are reactionary to the immediate past is that towards increased simplicity in the exercises of the various topics. This has been particularly pronounced in the last two decades, with the result that the percentage of complicated exercises in every topic is now generally less than in the decades from 1880 to 1909.

The present reaction towards simplicity is for most topics merely the renewal of a tendency that began early in the nineteenth century and that was intermittently present in that century. It is, in fact, the last of a series of three reactions against the complexity of algebraic exercises. The percentages of complicated exercises in equations, roots and radicals, addition and subtraction, fractions, multiplication, and division were appreciably less in the second decade than in the first. This change was perhaps too great or too sudden to continue. Whatever the reason, the tendency toward increased simplicity did not last much more than a decade. The percentage of complicated exercises gradually increased in most of the topics in the succeeding decades. In the seventies there was a renewal, or reappearance, on a lesser scale, of the tendency of the thirties. In the decade, 1870–1879, the percentages of simple exercises increased between five and thirteen per cent in eight of the ten topics that had been increasing in complexity. This second reaction was also temporary and in the following decade the percentage of complicated exercises increased in all topics of the course with the exception of addition and subtraction.

The third or present expression of the tendency towards increased simplicity first appeared in the last decade of the nineteenth century in equations, division, progressions, and common factors and multiples. In the remainder of the topics it appeared a decade or two later. It did not, however, become pronounced sufficiently to change materially the percentages of complicated exercises until the second decade of the present century.

The present tendency towards increased simplicity in factoring and parentheses is a reversal of the trend preceding the last two or three decades. It has resulted in a return in part to the percentages customary in the middle decades of the last century. Contrary to the tendency in most of the topics the percentage of complicated exercises in highest common factor and lowest common multiple decreased steadily from the decade of the thirties, when common factors and multiples first appeared in the course, to that of the sixties. The percentage increased in the seventies and eighties. In the nineties, however, the original tendency to decrease reappeared. It has continued to the present, with the result that now the percentage of simple exercises in this topic is greater than at any other time in the period studied.

The present tendency to increase the percentage of exercises in certain topics and to decrease it in others has resulted at times in increased similarity to some of the percentages preceding the eighties. Among the topics of which this is true to a greater or less degree are equations, formulas, addition and subtraction, negative exponents, proportions, and factoring. Equations have tended to increase in percentage since the beginning of the present century. This is contrary to the tendency generally prevailing throughout the last century. The result is that the percentage of exercises in equations is now more similar to the percentages in the middle decades of the past century than to the last three decades of that century. In the first fifty years more than one exercise in three was an equation. At present the ratio is not yet as high as this. The percentage of equations now amounts to 30.99. In formulas the percentage now is not only most similar to the percentages in the twenties and fifties, but the tendency to increase, which has been evident in the last three decades, is paralleled by a similar tendency in the twenty years preceding 1860. The stress, as now, was generally in the application rather than the derivation of formulas.

The changes in the stress given these two topics (equations and formulas) and the other topics mentioned above may be termed reactionary. They cannot be termed revolutionary or radical in the sense of being new or extreme, although in some

cases the percentages have not only reached those of the earlier nineteenth century, but have passed them. In other topics the percentages have changed very little. At present, fractions and division, both of which tended generally to decrease in the nineteenth century, show little tendency to change from the percentage of 1890. Multiplication has varied only slightly since 1830. Parentheses increased from .23 per cent in the first decade to 2.18 in the decade from 1910 to 1919. The present percentage, 1.96, is slightly lower than that for the decade preceding it, but the decrease is so small that it is impossible to say whether or not this topic is actually lessening in importance.

The changes in percentage or relative stress laid on the more specific phases of the general topics are discussed in the chapter for each topic. Briefly, they result from tendencies that are in some cases evolutionary, in other cases reactionary. In only a few cases have the changes neither precedents in changes occurring in preceding decades nor points of similarity to the conditions existing in those decades.

Part C

Comparison with the Report of the National Committee on Mathematical Requirements

The report of the National Committee on Mathematical Requirements, entitled the *Reorganization of Mathematics in Secondary Education*, was first published in 1923. It is the work of a committee, organized in 1916 under the auspices of The Mathematical Association of America, "for the purpose of giving national expression to the movement for reform in the teaching of mathematics, which had gained considerable headway in various parts of the country, but which lacked the power that coordination and united effort alone could give."[1] It is, therefore, to the point to compare the findings in the present study with the recommendations made in this report. It must be understood, however, that comparisons are only possible with one phase

[1] *The Reorganization of Mathematics in Secondary Education* (*Part 1*), Houghton Mifflin Co., 1927. Page vii.

of the report of the committee. The *Reorganization of Mathematics in Secondary Education* is an extensive investigation and in it are made recommedations for the content, organization, and method of various branches of mathematics. This study deals only with elementary algebra from the viewpoint of its content. That a more or less arbitrary separation of content from organization and method and of elementary algebra from the other branches of mathematics has been made in this study must be kept in mind in making comparisons.

The viewpoint of the National Committee in regard to the content of elementary algebra is expressed in the two following quotations:

"Continued emphasis throughout the course must be placed on the development of ability to grasp and to utilize ideas, processes, and principles in the solution of concrete problems rather than on the acquisition of mere facility or skill in manipulation. The excessive emphasis now commonly placed on manipulation is one of the main obstacles to intelligent progress. On the side of algebra, the ability to understand its language and to use it intelligently, the ability to analyze a problem, to formulate it mathematically, and to interpret the result must be dominant aims. *Drill in algebraic manipulation should be limited to those processes and to the degree of complexity required for a thorough understanding of principles and for probable applications either in common life or in subsequent courses which a substantial proportion of the pupils will take.* It must be conceived throughout as a means to an end, not as an end in itself. Within these limits, skill in algebraic manipulation is important, and drill in this subject should be extended far enough to enable students to carry out the essential processes accurately and expeditiously."[2]

". . . much of the emphasis now generally placed on the formal exercise should be shifted to the 'concrete' or 'verbal' problem."[3]

If it is assumed that in the above quotations the committee had in mind the conditions existing in algebra in the decades immediately preceding its report, *i.e.*, 1890, 1900, and 1910, it may be inferred that the committee favors among other things the following changes in the conditions then existing:

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[2] *Op. cit.*, pp. 14–15.
[3] *Op. cit.*, p. 38.

1. An increase in both number and percentage of verbal exercises.
2. A decrease in both number and percentage of tool exercises.
3. A decrease in the emphasis on complicated algebraic technique.
4. A limiting of the drill to certain processes deemed necessary by the committee.

An Increase in Both Number and Percentage of Verbal Exercises

There has been a large change from the nineties in the mean number and percentage of verbal exercises, both in the direction[4] favored by the committee. Verbal exercises in the data for the present decade amount to 20.81 per cent. In the decade beginning in 1910, they totalled 18.05 per cent and in the nineties, 12.78 per cent. The present percentage represents, therefore, a gain of 8.03 points over the nineties. It is, however, still lower than the percentage of verbal exercises in any decade preceding 1860.

The mean number of verbal exercises per book has increased from 334.8 exercises in the nineties to 742.5 in the present decade. The number for the decade beginning in 1920 is, therefore, more than twice as large as the number for the decade of the nineties. Much of this increase took place in the decade from 1910 to 1919, when the mean number of verbal exercises increased 60.3 per cent over the decade from 1900 to 1909. The increase of the present decade over the decade beginning in 1910 amounts to 31.3 per cent. At present there is slightly more than one verbal exercise for every four tool exercises. In the nineties the ratio was approximately one to six. The present ratio, however, is less than that in the decades preceding 1860, which was approximately one to three.

Therefore, it may be said that the changes that have taken place in the relative stress laid on verbal exercises in the last thirty years have been in the direction recommended by the National Committee.[4] The changes, however, have not been

[4] It cannot be inferred from this, however, that these changes have necessarily led to conditions desired by the committee. The report does not indicate how far changes should be carried. Present changes, although in the direction recommended, may consequently have fallen short of, reached, or surpassed the limits intended by the committee.

great enough to lead to as much emphasis in verbal exercises as in the earlier decades of the nineteenth century.

A Decrease in Both Number and Percentage of Tool Exercises

Tool exercises decreased from 82.69 per cent in the last decade of the nineteenth century to 74.78 per cent in the present decade. In this respect the present change in tool exercises is in accordance with the committee's recommendations.[5] The decrease was greatest in the decade from 1910 to 1919. Nevertheless, the present figure is still higher than the percentages customary in the interval between 1830 and 1879.

The mean number of tool exercises, however, has continued to increase. In the decade of the nineties, it amounted to 2166.8 exercises and in the decade beginning in 1910, to 2424.3 exercises. In the present decade it amounts to 2667.9 exercises, an increase of approximately 10 per cent over the decade from 1910 to 1919 and of 23 per cent over the decade of the nineties. This continued increase in tool exercises is apparently contradictory to the recommendations made by the National Committee, unless such increase is necessary to get the skill in algebraic manipulation deemed essential by them. (See first quotation, page 158.)

In conclusion, therefore, it may be said that tool exercises have decreased in percentage but increased in mean number in the interval from 1890 to the present. The former is apparently in accord with the committee's recommendations,[6] the latter is not.

A Decrease in the Emphasis on Complicated Algebraic Technique

There has been a decrease in the percentage of complicated exercises in every topic of the elementary course in the last thirty years. The decrease began in some topics in the decade of the nineties, but in most topics in the following decade. It was especially pronounced in the decade from 1910 to 1919.

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[5] Assuming that the changes have not exceeded the limits desired by the committee. See Note 4.

[6] See Notes 4 and 5.

The present reaction to the complexity of the exercises in elementary algebra is more pronounced and persistent than the reaction that took place in the decades of the thirties and the seventies. The result is that now the percentage of complicated exercises in equations, roots and radicals, multiplication, division, common factors and multiples, and addition and subtraction is less than in any other decade since algebra became a high school subject. Although the percentages of complicated exercises in fractions, parentheses, and factoring are still higher than in the decades preceding 1860, they are materially lower than in the latter decades of the nineteenth century. If the present rate of decrease continues, these topics will have as large a percentage of simple exercises in a decade or two as they had before.

This tendency towards elimination of much of the complicated drill in algebraic topics is along the line recommended by the Committee.

A Limiting of the Drill to Certain Processes Deemed Necessary by the Committee

Whether or not the drill in algebraic manipulation has been limited to those processes that the committee recommends is best seen by comparing the percentage of drill in those topics with the percentage of drill in the topics recommended for omission or decrease. The formula, graph, positive and negative numbers, and equation are the topics most stressed by the committee. These have all increased in percentage recently. Formulas now have a larger percentage—14.73—of drill than ever before. This is true also of graphs, which are a recent addition to the elementary course, as judged by the textbooks examined and which now amount to 3.60 per cent of the total exercises. Equations and addition and subtraction of positive and negative numbers are now given more emphasis than in the closing decades of the last century, but not so much as was common in the earlier decades of that century.

Factoring, common factors and multiples, roots and radicals, negative exponents, simultaneous and radical equations, and the

binomial theorem, have, on the contrary, decreased more or less in percentage of drill in the years since 1890. These topics were recommended for omission or decrease in importance by the committee. Therefore, the present changes are in harmony with the suggestions of the committee.[7]

In conclusion, therefore, it may be said that the percentages in the various topics are more in accord with the viewpoint of the Committee[7] than are the percentages of thirty or forty years ago. The changes in these percentages began in some cases many decades ago; in most topics, however, the tendency to change started at the beginning of this century. Frequently the changes result from the renewal of tendencies appearing in the earlier decades of the past century, and frequently they result in similarities to conditions existing in the earlier decades of the period.

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[7] See Notes 4 and 5

APPENDIX

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